Master Sciences de la Matière **ENS Lyon**
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Travaux Dirigés de Physique Nucléaire

Form factors : introduction, basic properties, nuclear radius

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These exercises should allow to accustom yourselves with elementary properties of form factors. In particular, they should give you an idea about some orders of magnitude. We will also present a way to estimate the value of nuclear radii using elementary electrostatics.

We recall the long distance behavior of the wave function after a scattering on a potential $V(r)$:

$$\psi(\vec{r}) \simeq \mathrm{e}^{ikz} + f(\theta,\varphi)\frac{\mathrm{e}^{ikr}}{r} + \mathrm{o}\left(\frac{1}{r}\right)$$

The above formula is normalized in such a way that the flux of incoming particles is 1. This flux is along the z axis and the scattering center is located at $\vec{r} = 0$ and generates the scattering potential $V(r)$. Finally, (r, θ, φ) are the usual spherical coordinates with respect to the scattering center and the Oz axis.

1 / Show that the differential cross section reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \mid f(\theta,\varphi) \mid^2 \quad .$$

Now take into account that the scattering center is not a point but occupies a certain small region of space (small with respect to what ?) with density ρ and comment about the origin of the form factor.

2 / Show that the q^2 derivative of a nuclear form factor at $q = 0$ is proportional to $\langle(\vec{r}.\widehat{q})^2\rangle$, where $\widehat{q}$ stands for the direction of the momentum transfer.

3 / What is the form factor of a homogeneous sphere of radius a ? Choose a good dimensionless parameter to express the form factor. What are the three first zeros of $F(q)$? Why is it interesting to consider such form factors ?

4 / Use the plot on Figure 1 to estimate the nuclear radius of ^{14}N. What is the value of the scattering angles corresponding to the three first zeros of $F(q)$? What should be the angular resolution of the detecting device so that the relative uncertainty on the measurement of the radius is of the order of 10% ? Which minimum of the form factor is best fit for a measurement of the nuclear radius ?

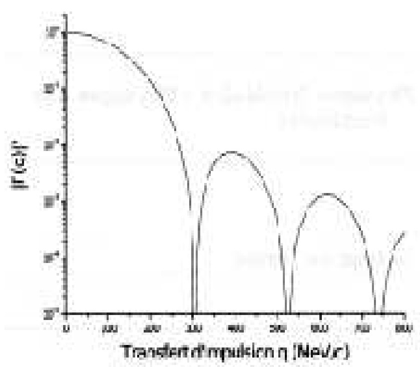

FIGURE 1 – Form factor of ^{14}N as a function of the transferred momentum during the scattering.

5 / The Rutherford cross section isn't anymore correct at high energies. There, it should be replaced by the relativistic Mott formula taking into account the spin and relativistic behavior of the scattered particles.
It reads :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = (Z_p Z_c \alpha \hbar c)^2 \frac{E'}{E} \frac{1 - \beta^2 \sin^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \ .$$

Z_p and Z_c are resp. the atomic numbers of the projectile and the target. E et E' are the final and initial energies of the projectile. Finally $\beta = v/c$, v being the projectile's velocity.

We have presented on Figure 2 the measurement of the differential cross section of electrons on H_2 versus the scattering angle. Use this plot to obtain the energy of the scattered electrons. Is one able to see, at such energies, the first zero of the form factor ?

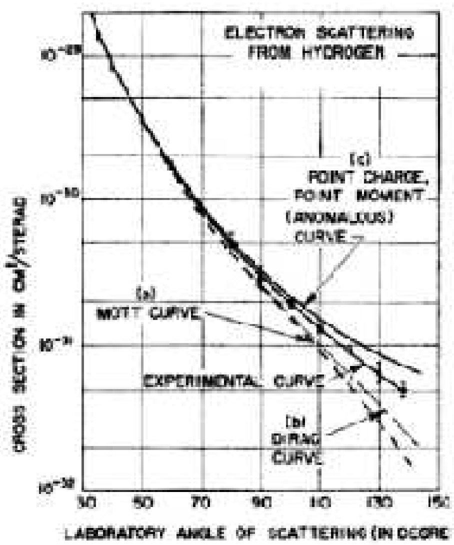

FIGURE 2 – Differential cross section of electrons on H_2 versus the scattering angle θ.

6 / The authors of the experiment have assumed that the density inside a proton is of the type

$$\rho(r) = \rho_0 \frac{\mathrm{e}^{-\frac{r}{a}}}{r}$$

Relate ρ_0 to the other constants. Compute the form factor. The experiment that gave rise to figure 2 allowed to measure the average square radius of the proton : $\langle r^2 \rangle^{1/2} = 0,74 \pm 0,24$ fm.

Mirror nuclei

7 / We are going to perform a classical estimate for the difference of energies between mirror nuclei (same A but different atomic number). The quantum computation of the same effect gives a result different by a couple of percents. The idea is to assume that mirror nuclei have approximatively the same radius and that the only difference in their energies is due to the presence of an additional coulomb interaction of the proton. Justify and discuss!

8 / What reasonable model could we take for the charge distribution inside of the nucleus? For the nuclear shape? Compute the electric potential and the corresponding energy in the case of Z protons.

9 / What is then the difference in energies between two mirror nuclei? In the case of ${}_{15}P^{29}$ and ${}_{14}S^{29}$, one measures $\Delta E = 4.96$MeV. Deduce the value of r_0 in the empirical formula for the radius.