Master Sciences de la Matière **ENS Lyon**
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Travaux Dirigés de Physique Nucléaire

Liquid drop model: binding energy, isobaric chains

Exercise 1: Energy balance

1 / Find the inequalities between the masses of the neutral "father" and "son" atoms in the case of:

- a β^- decay?
- a β^+ decay?
- an electron capture by the K level?

2 / What would be the experimental signatures?

Exercise 2: Binding energy per nucleon
Starting from the empirical spherical mass formula and neglecting the pairing term, show that the binding energy per particle (B/A) for symmetric nuclei $N = Z = A/2$ has a maximum close to the iron ($Z = 26$). Estimate the value at the maximum.
Numerical values: $u_s = 18\,\text{MeV}$, $u_c = 0.7\,\text{MeV}$ and $u_v = 15.8\,\text{MeV}$.

Exercise 3: Nucleus stability
1 / One considers a nucleus ${}^A_Z X$ with $A \gg 1$ and $Z \gg 1$. Express the Q-values of the following decays with respect to the binding energy and its derivatives:

- neutron emission
- proton emission
- α-particle emission
- symmetric fission ($A \longrightarrow A/2$ and $Z \longrightarrow Z/2$)

2 / Deduce the stability of a ${}^{238}_{92}U$ nucleus w.r.t. those various decay modes and compute the released energy when the decay is possible. The pairing term will be neglected. Numerical values: $u_s = 16.8\text{MeV}$, $u_v = 15.5\text{MeV}$, $u_T = 19\text{MeV}$, $u_c = 0.72\text{MeV}$ and $B_\alpha = 28\text{MeV}$.

Exercise 4: Neutron stars

1 / Extrapolate the mass formula by adding a gravitational potential energy term to study the stability of a neutron star. The surface and pairing terms will be neglected w.r.t. the volume term.

2 / What conditions should fulfil the radius and the mass of the neutron star? More accurate calculations give a limit mass of 0.1 $M_\odot$. Comment.
($m_n c^2 = 1.67\ 10^{-27}\text{kg}$ and $G = 6.673\ 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$, $M_\odot \sim 2\ 10^{30}\,\text{kg}$).

Exercise 5: Odd isobaric chain

For the isobaric chain $A = 151$ one gets the following mass excesses:

Z	Element	Mass excess
58	Ce	-62.620
59	Pr	-67.160
60	Nd	-70.957
61	Pm	-73.400
62	Sm	-74.587
63	Eu	-74.663
64	Gd	-74.198
65	Tb	-71.632
66	Dy	-68.902
67	Ho	-63.803
68	Er	-58.500
69	Tm	-51.000
70	Yb	-41.100
71	Lu	-29.010

A	Z	Element	Mass excess
1	1	H	+7.289
4	2	He	+2.424
147	69	Tm	-34.820
150	70	Yb	-37.470

1 / Identify the stable isobar(s) of that serie.

2 / Give the decay modes of the unstable isobar(s). Summarize the results on a $M(Z)$ scheme.

3 / For ^{151}Lu indicate all possible decay modes.

4 / In which Z domain should one look for spontaneous neutron emission in that isobaric chain?

Exercise 6: Even isobaric chain

The following nuclei belong to the $A = 114$ isobaric chain:

$_{46}$Pd $_{47}$Ag $_{48}$Cd $_{49}$In $_{50}$Sn $_{51}$Sb $_{52}$Te

where

- $_{46}$Pd is a β^- emitter with 1.4 MeV maximal energy of the electrons;
- $_{47}$Ag gives a β^- of 4.6 MeV maximal energy, followed by a gamma emission of 0.56 MeV;
- $_{51}$Sb gives two β^+, the first one of 2.7 MeV maximal energy followed by a two-gamma cascade of 0.9 MeV and 1.3 MeV, the second one with 3.6 MeV maximum energy followed by a 1.3 MeV gamma.

Find for that particular chain:

1 / the coefficients of the mass formula (parabolic form),

2 / the stable nucleus (nuclei),

3 / the various decay modes of unstable isobar(s) and the maximum energy of the emitted β.