
Travaux Dirigés de Physique Nucléaire

Decay modes, α radioactivity, simple Gamov theory

#1: Potential barrier

We consider the one-dimensional propagation of a particle through a square potential barrier of width a and height V_0 given by

$$V(x) = \begin{cases} 0 & \text{if } x < 0 & \text{region I,} \\ V_0 & \text{if } 0 < x < a & \text{region II,} \\ 0 & \text{if } a < x & \text{region III,} \end{cases}$$

A travelling wave of energy E and wave number k_1 , originating from $x \rightarrow -\infty$, enters regions II and III without undergoing any reflection in region III.

1 / Using the one-dimensional Schrödinger equation and suitable boundary conditions at the frontiers $x = 0$ and $x = a$, estimate the transmission and reflection coefficients of the wave in the case $E > V_0$.

2 / How are modified the previous solutions in the case $E < V_0$ (tunnel effect) ? use the notation:

$$\rho = \left(\frac{2m}{\hbar^2} (V_0 - E) \right)^{1/2}$$

3 / Such a potential barrier is said “thick” if $\rho a \gg 1$. Give an approximation to the transmission coefficient $T(E)$ in this case, of the type

$$T(E) = \exp(-2 a \rho) [..],$$

and give the value of the coefficient [..].

4 / Arbitrary barrier. One now considers the case where the barrier is not square but still has a maximum larger than E . Let $x_{1,2}$ be the abscissas of classical turning points, fulfilling: $V(x_1) = V(x_2) = E$. Show by heuristic arguments, in particular by splitting the barrier into “slices” of width dx for which the previous approximation is assumed to hold, that a plausible expression of the transmission coefficient $T(E)$ reads:

$$T(E) = \exp \left[-2 \int_{x_1}^{x_2} \rho(x) dx \right].$$

#1: α Radioactivity. Gamov theory

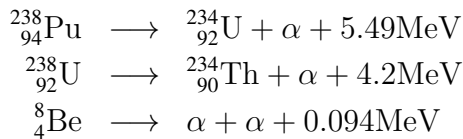
A theory by Gamov describes the α -disintegration probability as a product of 3 terms:

- probability of formation of an α cluster in the nucleus,
- number of collisions of the α at the nucleus surface,
- probability of tunnelling through the potential barrier.

1 / Explain why the collision term is of the order of $v_\alpha / (2R)$, v_α being the speed of the emitted α . Estimate this term for a nucleus of mass

$150 < A < 250$ emitting α of average energy $4 < E_\alpha < 7$ MeV. Show that this term is about $\sim 10^{21}$.

Estimate numerically $v_\alpha/(2R)$ for the following decays:



2 / The effective α -nucleus potential, $V(x)$, where x is the distance between the α cluster and the nucleus, is modelled for $x \leq R$ by a square well of radius R and height $V = 2Ze^2/(4\pi\epsilon_0 R)$, insuring continuity with the repulsive Coulomb interaction $2Ze^2/(4\pi\epsilon_0 x)$ which holds for $x \geq R$. Draw a graphical representation of this potential.

3 / One is now interested in the probability of tunnelling through this barrier. Let R' denotes

the classical turning point, i.e., $V(R') = E_\alpha$. Calculate

$$I = \int_R^{R'} dx \left(\frac{8m}{\hbar^2} (V(x) - E_\alpha) \right)^{1/2}$$

4 / Show that if $R \ll R'$, the following approximation is obtained:

$$I \simeq \left(\frac{8mZze^2}{\hbar^2} \right)^{1/2} \left(\frac{\pi}{2} \sqrt{R'} - 2\sqrt{R} \right)$$

5 / Calculate the radius R as a function of the mass number of the daughter nucleus and the α particle.

6 / Estimate *numerically* the α -lifetimes $\tau({}^{238}\text{Pu})$ et $\tau({}^8\text{Be})$ and compare to the experimental values:

$$\tau({}^8\text{Be}) = 2.6 \cdot 10^{-17} \text{ s}, \quad \tau({}^{238}\text{Pu}) = 128 \text{ years.}$$

It is recalled that: $\hbar c = 197 \text{ MeV}\cdot\text{fm}$.