Master Sciences de la Matière **ENS Lyon**
Année 2013-14

Travaux Dirigés de Physique Nucléaire

Decay modes, α radioactivity, simple Gamov theory

#1: Potential barrier

We consider the one-dimensional propagation of a particle through a square potential barrier of width a and height V_0 given by

$$V(x) = \begin{cases} 0 & \text{if} \quad x < 0 \quad \text{region I}\,, \\ V_0 & \text{if} \quad 0 < x < a \quad \text{region II}\,, \\ 0 & \text{if} \quad a < x \quad \text{region III}\,, \end{cases}$$

A travelling wave of energy E and wave number k_1, originating from $x \to -\infty$, enters regions II and III without undergoing any reflection in region III.

1 / Using the one-dimensional Schrödinger equation and suitable boundary conditions at the frontiers $x = 0$ and $x = a$, estimate the transmission and reflection coefficients of the wave in the case $E > V_0$.

2 / How are modified the previous solutions in the case $E < V_0$ (tunnel effect) ? use the notation:

$$\rho = \left(\frac{2m}{\hbar^2}(V_0 - E) \right)^{1/2}$$

3 / Such a potential barrier is said "thick" if $\rho\, a \gg 1$. Give an approximation to the transmission coefficient $T(E)$ in this case, of the type

$$T(E) = \exp(-2\, a\, \rho)\, [..]\,,$$

and give the value of the coefficient [..].

4 / Arbitrary barrier. One now considers the case where the barrier is not square but still has a maximum larger than E. Let $x_{1,2}$ be the abscissas of classical turning points, fulfilling: $V(x_1) = V(x_2) = E$. Show by heuristic arguments, in particular by splitting the barrier into "slices" of width $\mathrm{d}x$ for which the previous approximation is assumed to hold, that a plausible expression of the transmission coefficient $T(E)$ reads:

$$T(E) = \exp\left[-2 \int_{x_1}^{x_2} \rho(x) dx \right]\,.$$

#1: α Radioactivity. Gamov theory

A theory by Gamov describes the α-disintegration probability as a product of 3 terms:

- probability of formation of an α cluster in the nucleus,
- number of collisions of the α at the nucleus surface,
- probability of tunnelling through the potential barrier.

1 / Explain why the collision term is of the order of $v_\alpha/(2R)$, v_α being the speed of the emitted α. Estimate this term for a nucleus of mass

$150 < A < 250$ emitting α of average energy $4 < E_\alpha < 7$ MeV. Show that this term is about $\sim 10^{21}$.
Estimate numerically $v_\alpha/(2R)$ for the following decays:

$$\begin{aligned} {}^{238}_{94}\mathrm{Pu} &\longrightarrow {}^{234}_{92}\mathrm{U} + \alpha + 5.49\mathrm{MeV} \\ {}^{238}_{92}\mathrm{U} &\longrightarrow {}^{234}_{90}\mathrm{Th} + \alpha + 4.2\mathrm{MeV} \\ {}^{8}_{4}\mathrm{Be} &\longrightarrow \alpha + \alpha + 0.094\mathrm{MeV} \end{aligned}$$

2 / The effective α–nucleus potential, $V(x)$, where x is the distance between the α cluster and the nucleus, is modelled for $x \leq R$ by a square well of radius R and height $V = 2\,Ze^2/(4\pi\epsilon_0\,R)$, insuring continuity with the repulsive Coulomb interaction $2\,Ze^2/(4\pi\epsilon_0\,x)$ which holds for $x \geq R$. Draw a graphical representation of this potential.

3 / One is now interested in the probability of tunnelling through this barrier. Let R' denotes the classical turning point, i.e., $V(R') = E_\alpha$. Calculate

$$I = \int_R^{R'} \mathrm{d}x \left(\frac{8m}{\hbar^2} (V(x) - E_\alpha) \right)^{1/2}$$

4 / Show that if $R \ll R'$, the following approximation is obtained:

$$I \simeq \left(\frac{8mZze^2}{\hbar^2} \right)^{1/2} \left(\frac{\pi}{2}\sqrt{R'} - 2\sqrt{R} \right)$$

5 / Calculate the radius R as a function of the mass number of the daughter nucleus and the α particle.

6 / Estimate *numerically* the α-lifetimes $\tau(^{238}\mathrm{Pu})$ et $\tau(^{8}\mathrm{Be})$ and compare to the experimental values:

$$\tau(^{8}\mathrm{Be}) = 2.6\ 10^{-17}\,\mathrm{s}, \qquad \tau(^{238}\mathrm{Pu}) = 128\,\mathrm{years}.$$

It is recalled that: $\hbar\,c = 197$ MeV.fm.