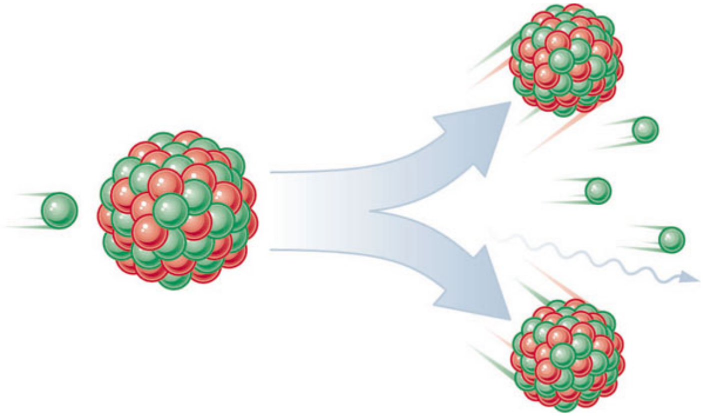
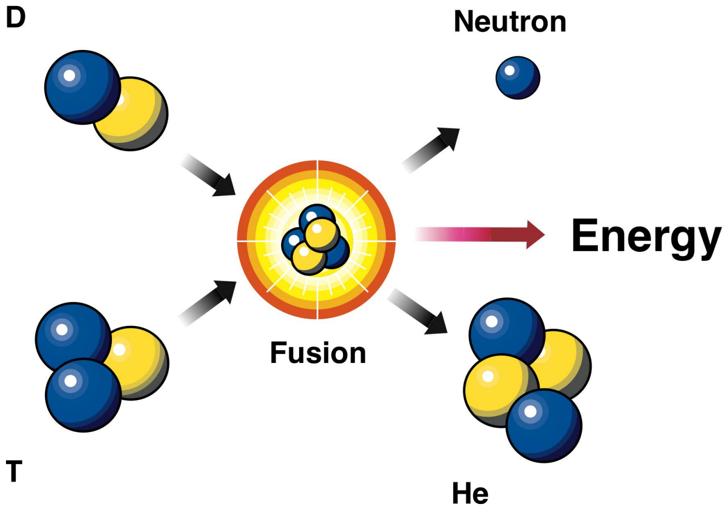
Chapter 7
Fission, fusion and nuclear astrophysics





Outline/Plan

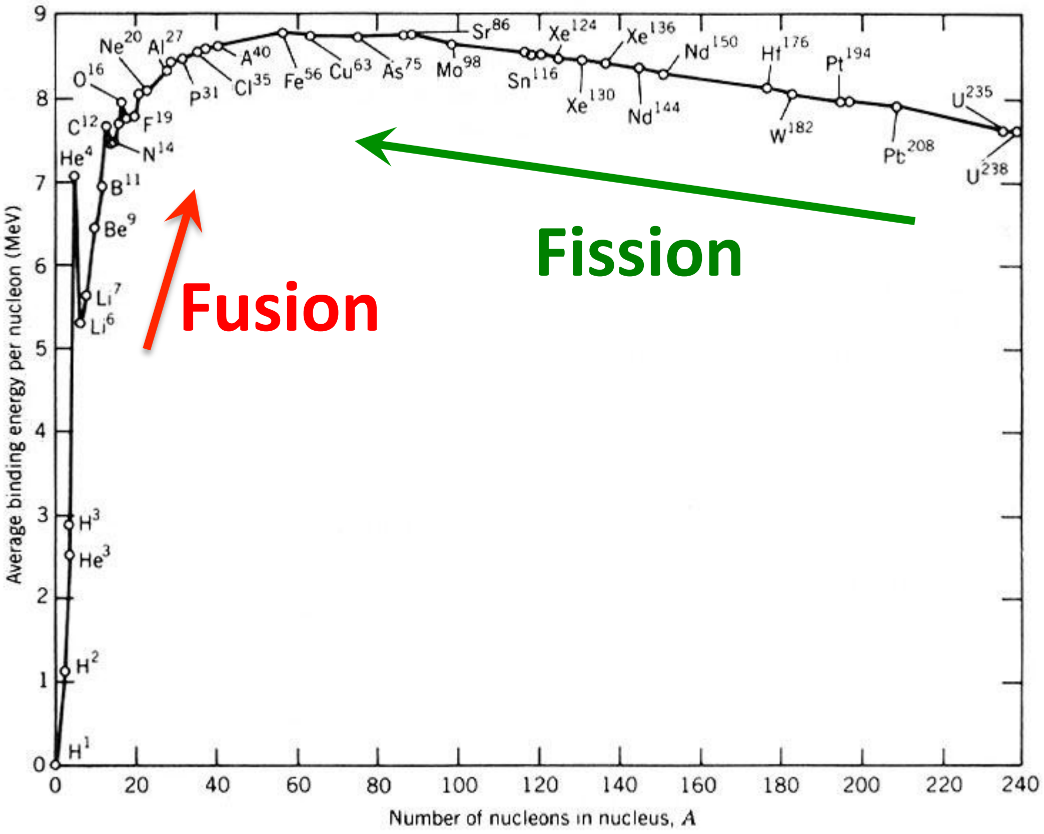
1. Fission
    1. Spontaneous fission
    2. Induced fission
    3. Chain reaction
    4. Fission reactor
2. Fusion
    1. Fusion in the sun
    2. Other fusion processes in stars
    3. Controlled fusion

1. Fission
    1. Fission spontanée
    2. Fission induite
    3. Réaction en chaine
    4. Réacteur à fission
2. Fusion
    1. Fusion dans le soleil
    2. Autres processus de fusion dans les étoiles
    3. Fusion contrôlée

Introduction

- Most stable form of nuclear matter is around A=60.
  B/A ≈ 8.5 MeV

- Energy is released
  - in the fission of a heavy nucleus into two medium-sized nuclei.
  - In the fusion of two light nuclei into a single medium-sized nucleus.

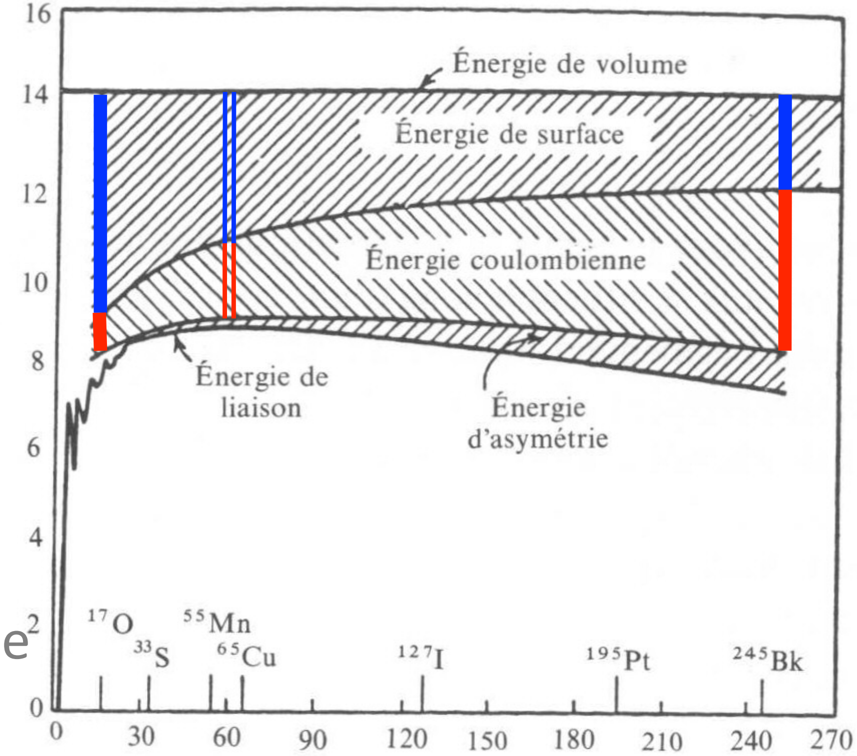


1- Fission

- Origin : competition between surface and coulomb energy

- Fission occurs because the total Coulomb repulsion energy of protons is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its effect is smaller.

- Fusion occurs because the low A nuclei have a too large surface area for their volume. The surface area decreases when they amalgamate. Coulomb energy increases in the process but its effect is smaller.



1.1- Spontaneous fission

Fission: $${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A_1}_{Z_1}\,\mathrm{Y} + {}^{A_2}_{Z_2}\,\mathrm{W}$$

- Expect spontaneous fission to occur if

$$Q_f = M(X) - M(Y) - M(W) = B(A_1, A_1) + B(A_2, Z_2) - B(A, Z) > 0$$

- Lets define: $y = \frac{A_1}{A} = \frac{Z_1}{Z}$ and $1 - y = \frac{A_2}{A} = \frac{Z_2}{Z}$ (the Z/N ratio is conserved)
- Then: $$Q_f = u_s A^{2/3}\left(1 - y^{2/3} - (1-y)^{2/3}\right) + u_c \frac{Z^2}{A^{1/3}}\left(1 - y^{5/3} - (1-y)^{5/3}\right)$$
- Maximum energy released when $\frac{\partial Q_f}{\partial y} = 0$

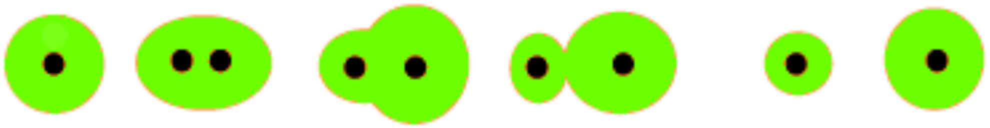
$$\frac{\partial Q_f}{\partial y} = \frac{2}{3} u_s A^{2/3}\left(-y^{-1/3} + (1-y)^{2/3}\right) + \frac{5}{3} u_c \frac{Z^2}{A^{1/3}}\left(-y^{2/3} + (1-y)^{2/3}\right) = 0$$

occurs when y=1/2

- For symmetric fission (y=1/2), the energy released is $Q_f^{\mathrm{sym}} = 0.37 u_c \frac{Z^2}{A^{1/3}} - 0.26 u_s A^{2/3}$
- Example: for ^{238}U, $Q_f \approx 200$ MeV ~ 10^6 × energy released in chemical reaction

Fission barrier

In order to fission, the system has to first go through a deformation process:

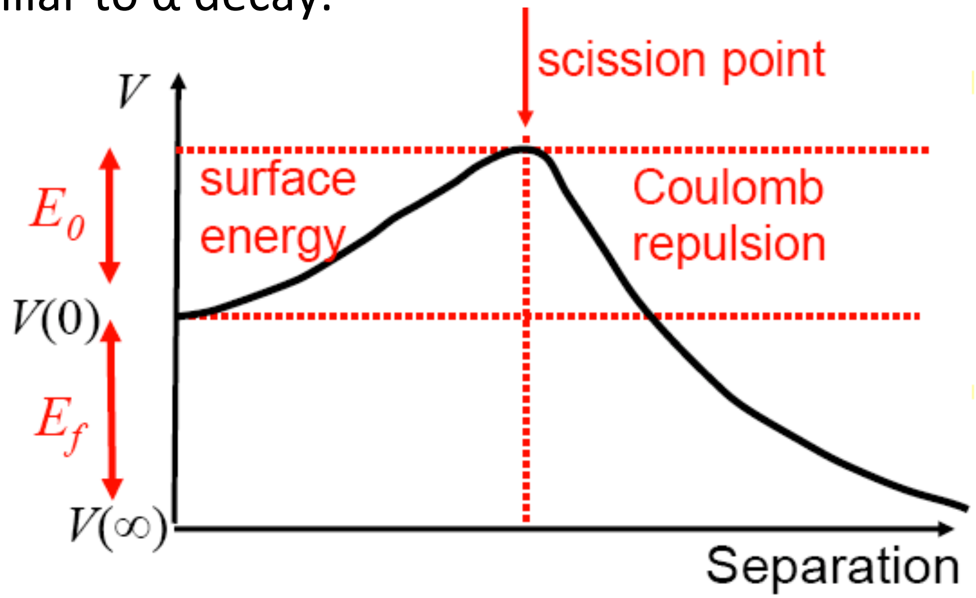


In the early stages of the deformation the surface energy is increased, but the coulomb energy is not yet much reduced. After the separation of the two fission fragments (at the scission point), the coulomb repulsion takes over.

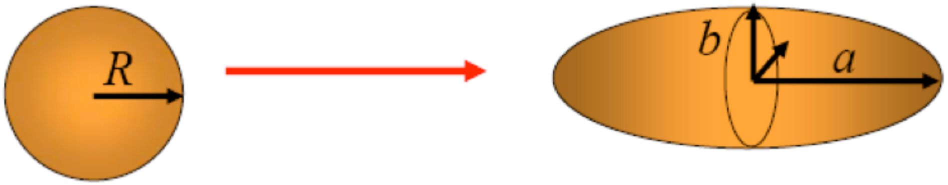
⇒During the fission process the system has to go through a barrier.
This is a tunneling problem similar to α decay.

E_f = energy released
→ Kinetic energy of fragments

E_0 = Fission activation energy
typically 6 MeV in ^{236}U



- Investigate the simplest deformation of the nucleus: ellipsoid shape in the liquid drop model



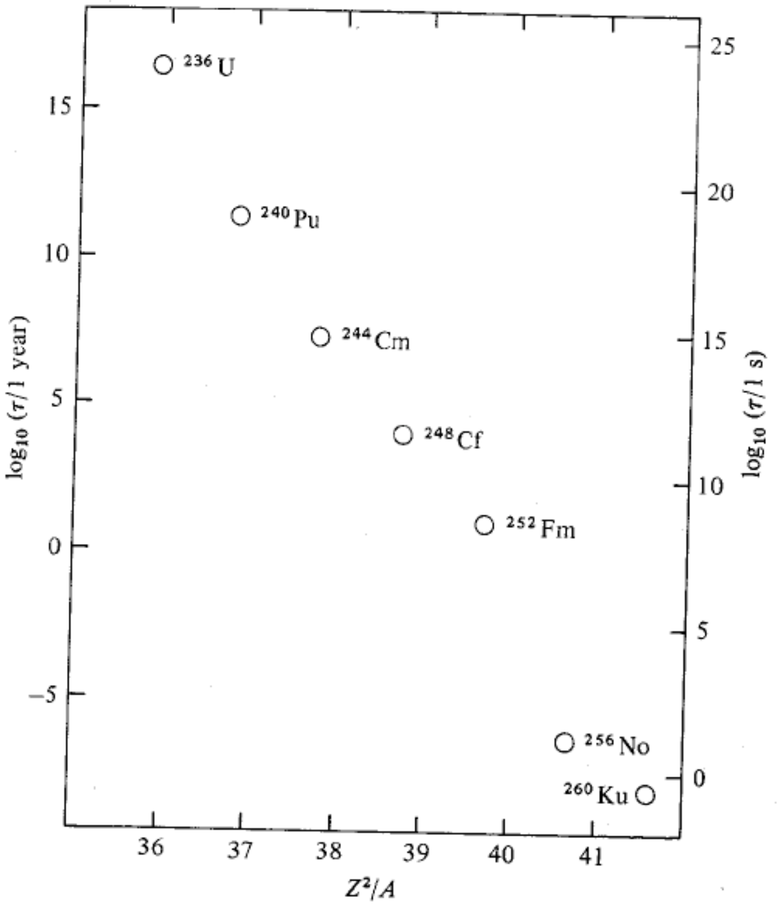
$$\begin{cases} a = R(1+\epsilon) \\ b = R(1+\epsilon)^{-1/2} \end{cases} \qquad \text{with } \epsilon \ll 1$$

- The volume term is unchanged (incompressible matter): $V = \frac{4}{3}\pi R^3 \Rightarrow \frac{4}{3}\pi a b^2$
- Change in surface term: $B_{\text{surf}} = -u_s A^{2/3} \Rightarrow -u_s A^{2/3}(1 + \frac{2}{5}\varepsilon^2))$ (geometry)
- Change in coulomb term: $B_{\text{coul}} = -u_c \frac{Z^2}{A^{1/3}} \Rightarrow -u_c \frac{Z^2}{A^{1/3}}(1 - \frac{1}{5}\varepsilon^2)$ (rather lengthy calculation)
- No changes in the asymmetry and pairing term.
- Binding energy of a deformed nucleus: $B_{\text{def}} = B_{\text{sph}} - \frac{2}{5}u_s\varepsilon^2 A^{2/3} + \frac{1}{5}u_c\varepsilon^2\frac{Z^2}{A^{1/3}}$
- A deformed shape is more stable than a spherical one if:

$$B_{\text{def}} > B_{\text{sph}} \Longrightarrow \frac{Z^2}{A} > \frac{2a_s}{a_c} \simeq 47$$

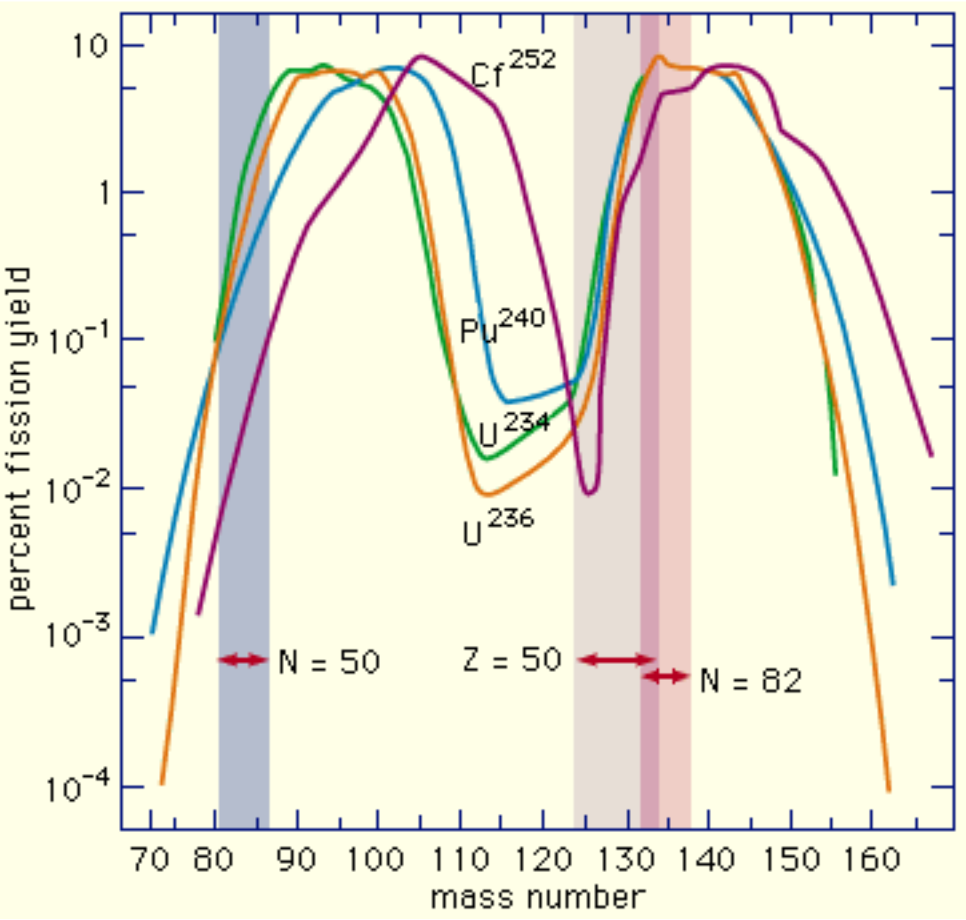
- Spontaneous fission criteria: $\frac{Z^2}{A} > 47$

Experimental results



Spontaneous fission lifetimes

Lifetime falls rapidly as Z^2/A increases

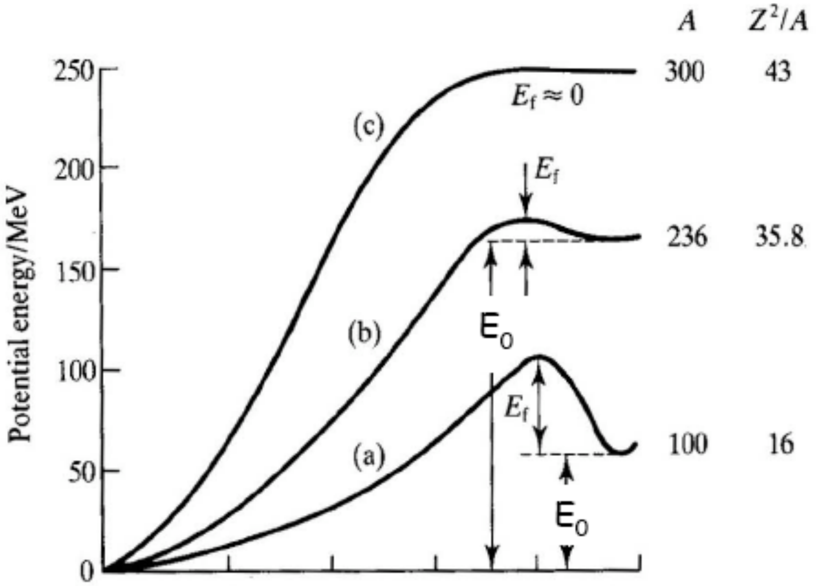


fission yield

Although liberated energy is maximal for symmetric fission, so is the coulomb barrier in fact fission is asymmetric

+ emission of neutrons

- The tunneling probability depends on Z^2/A
  → smaller activation energies for heavy nuclei



Evolution of fission barriers

- The tunneling probability depends on $\sqrt{m}$: $T =^{-2G}$ with $G \propto \sqrt{m}$
  → large mass : low tunneling probability
  Fission is much less probable than α decay

example for ^{238}U

$\lambda_\alpha = 5\ 10^{-18}\ s^{-1}$ → $T_{1/2}(\alpha) = 4.5\ 10^9$ yrs
$\lambda_{fi} = 3\ 10^{-24}\ s^{-1}$ → $T_{1/2}(\alpha) = 7.3\ 10^{15}$ yrs
fission is 10^6 less probable than α decay

Spontaneous fission is
a very rare decay mode
for stable or almost stable nuclei

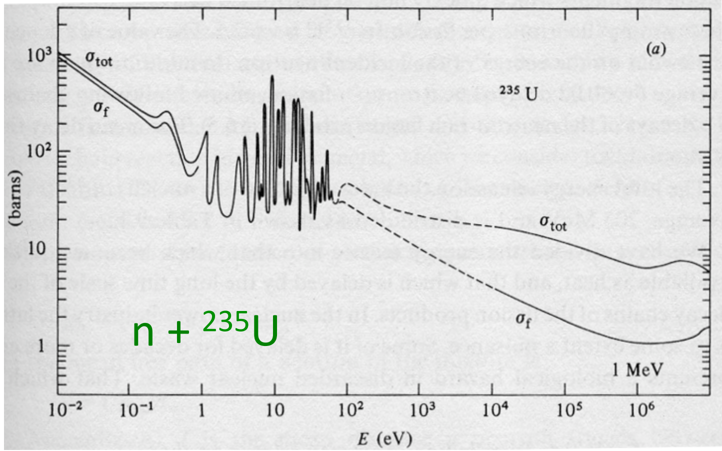
1.2 Induced fission

- Even at low energies, neutrons can be absorbed by nuclei (no coulomb barrier).

  Low energy neutrons can have very large absorption cross-sections

  apart from resonance

  $$\sigma(n,\gamma) \propto \frac{1}{v}$$



- After neutron capture, the compound nucleus is created in an excited state.
  - If the energy of the excited state is low (lower than the height barrier) γ decay is the most probable, but the excitation energy can help to increase the tunneling probability.
  - If the energy is about or greater than the fission activation energy, E_f fission is the dominant mode and will occur rapidly, even for zero energy neutron. → induced fission

Examples in Uranium

- In ^{236}U, the fission activation energy E_0 is about 6 MeV.
  A neutron absorption by ^{235}U, gives a compound nucleus ^{236}U* with an excitation energy greater than E_0 (even for zero energy neutron)
  → induced fission by thermal neutrons ($E_n \approx 1/40$ eV)

$$n + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{236}_{92}\mathrm{U}^* \longrightarrow \mathrm{X}^* + \mathrm{Y}^* + k\,n \qquad \text{with } k\approx 2.4$$

- In ^{238}U, the fission activation energy E_0 is also around 6 MeV

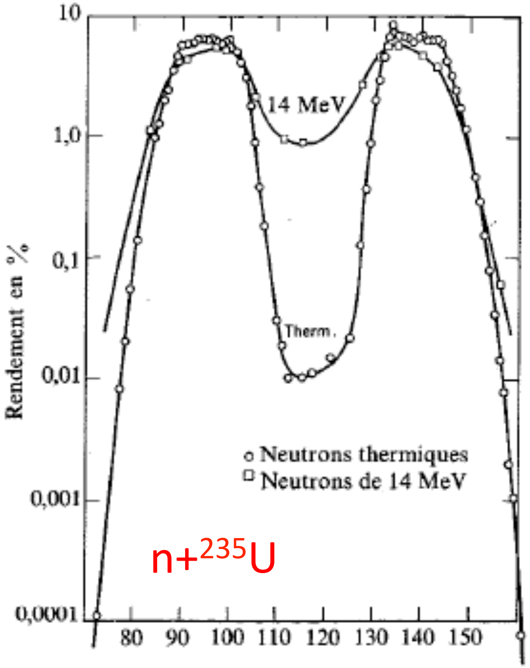
$$n + {}^{238}_{92}\mathrm{U} \longrightarrow {}^{239}_{92}\mathrm{U}^*$$

  but the excitation energy in ^{239}U is lower :

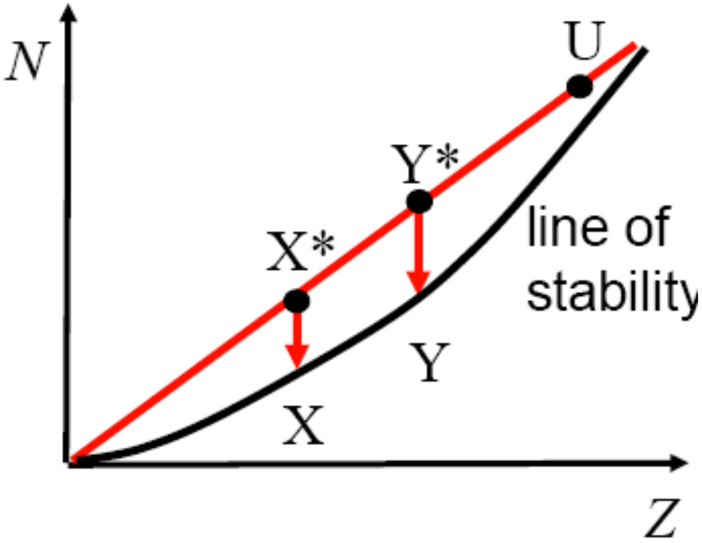
thermal neutrons	$E_n = 0.025$eV	$E^* \approx 5$ MeV	no thermal fission
rapid neutrons	$E_n > 1$ MeV	$E^* \approx 6$ MeV	rapid fission

- ^{235}U is a more interesting isotope for fission reactor

- Masses of fragments are unequal (in general).
  - Tend to have Z,N near magic number.
  - Less asymmetric distribution for rapid neutrons.
- Fragments X*, Y* tend to have the same Z/N ratio as the parent
  → neutron rich nuclei which emit prompt neutrons (10^{-16}s)
- X and Y undergo β decay mode more slowly (may also undergo neutron emission)
  → delayed neutron emission
  (~1 delayed neutron per 100 fissions)
- Wide variety of radioactive nuclei produced; potentially nasty
  → nuclear waste problem



n+^{235}U



1.3- Chain reaction

- Neutron from fission process can be used to induce further fission → chain reaction
- A chain reaction can be sustained if at least one neutron per fission induces another fission process

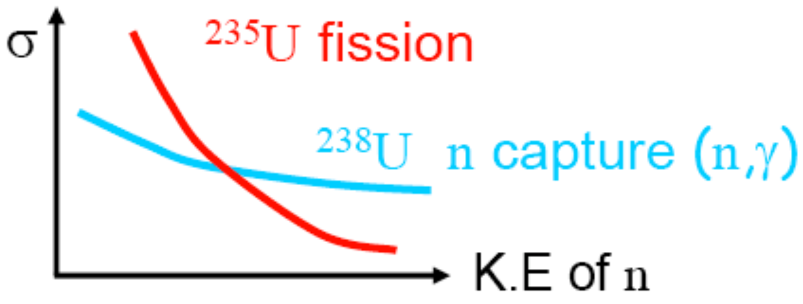
  let's define k the number of neutrons from one fission inducing another

k=1	**k<1**	**k>1**
Critical	Subcritical	Supercritical

- Prompt neutrons are fast $<E> \approx 2$ MeV → Their fission cross section is small. Need to slow down these neutrons before they can escape

1.4- Fission reactor

- Need a critical (k=1) chain reaction
- Need a moderator to slow down rapid neutrons via elastic collisions (large energy transfer). Requires a light nucleus (Carbon, heavy water D_2O,…)
- Combustible : problem, natural U is 99.3% ^{238}U and 0.7% ^{235}U, and n capture cross-section is large for ^{238}U. Again need to thermalise fast neutrons away from ^{238}U.



- Need to control the number of neutrons by absorption (use material with large neutron absorption x-section like boron, cadmium)

Overview of radioactive waste

Medium lived fission product

	$T_{1/2}$ (yr)	Yield(%)	Q (keV)
^{155}Eu	4.76	0.08	252
^{85}Kr	10.76	0.22	687
^{113m}Cd	14.1	0.01	316
^{90}Sr	28.9	4.51	2826
^{137}Cs	30.23	6.34	1176
^{121m}Sn	43.9	0.01	390
^{151}Sm	90	0.53	77

Long lived fission product

	$T_{1/2}$ (10^6 yr)	Yield(%)	Q (keV)
^{99}Tc	0.211	6.13	294
^{126}Sn	0.230	0.11	4050
^{79}Se	0.295	0.04	151
^{93}Zr	1.53	5.45	91
^{135}Cs	2.3	6.91	269
^{107}Pd	6.5	1.25	33
^{129}I	15.7	0.84	194

- The faster a radioisotope decay, the more radioactive it will be
- The chemical properties of the radioactive element will determine how mobile the substance is, and how likely it is to spread in the environment.

- What happens if the neutron control fails ?

  Let's define *N(t)*, the number of neutron at time t, *(k-1)* the change in number of neutron in on cycle, and τ(=10^{-3}) the mean time for one cycle (fission→fission)

$$N(t+\mathrm{d}t) = N(t) + (k-1)N(t)\frac{\mathrm{d}t}{\tau}$$

$$\Longrightarrow \mathrm{d}N = (k-1)N\frac{\mathrm{d}t}{\tau} \qquad \Longrightarrow \int_{N(0)}^{N(t)} \mathrm{d}N = (k-1)N\frac{\mathrm{d}t}{\tau}$$

$$\boxed{N(t) = N(0)e^{(k-1)t/\tau}}$$

Exponential growth

If *k*=1.01, $\tau=10^{-3}$s, after *t*=1s , $N(t)/N(0)=e^{10}\approx 22000$

Note: U reactor will not explode if it goes supercritical. As it heats up, the kinetic energy of neutrons increases and fission cross section drops. Reactor stabilizes at a very high temperature. ⇒Meltdown

2- Fusion

- Origin : competition between surface and coulomb energy

- Fission occurs because the total Coulomb repulsion energy of protons is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its effect is smaller.

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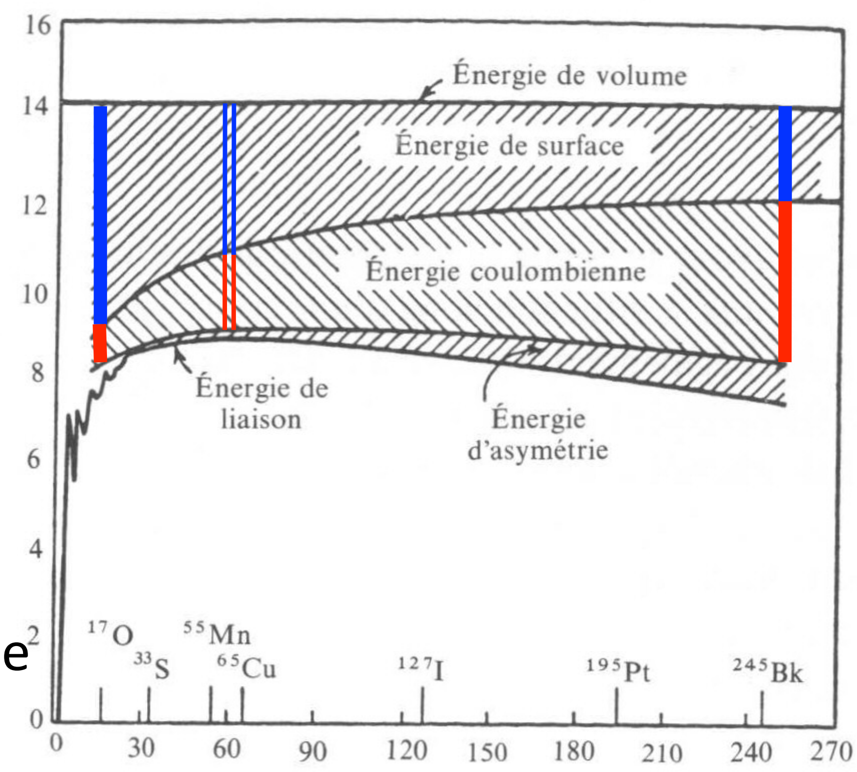


- It is energetically favorable for light nuclei to fuse and release energy. However nuclei need energy to overcome the coulomb barrier.
- The most basic process is : $p + p \longrightarrow d + e^+ + \nu_e$ $E_f = 0.42$ MeV

in this case the coulomb barrier is: $V = \frac{\alpha\hbar c}{r_0} = \frac{197}{137 \times 1.2} = 1.2MeV$

- Energies above the coulomb barrier are easy to achieve in accelerators. However, reaching high particle densities for long period of time is very difficult. This is a requirement to get a useful rate of fusion reactions for power generation
- In the stars, there is a large proton density (10^{32} m^{-3}), and these protons have kinetic energy due to thermal motion.

Obtaining $kT \approx 1$MeV require $T \approx 10^{10}$ K
In the interior of the sun, $T \approx 10^7$K, i.e. $\langle kT \rangle \approx 1$ keV

⇒Quantum mechanical tunneling required

2.1 Fusion in the sun

- Particles in the sun follow the Maxwell-Boltzman velocity distribution.
- Tunneling probability is a strong function of energy (i.e. velocity).
  → Tails of the Maxwell-Boltzman distribution are important.
- Reaction rate per unit of volume for particles of velocity v : $\Gamma = \sigma(v)\Phi N$
  where the flux is $\Phi = Nv$
- σ depends on the tunneling probability : $T = e^{-2G(v)}$
  with the Gamow factor:

$$G(v) \simeq \sqrt{\frac{2m}{E_0}} Z_1 Z_2 \alpha c \frac{\pi}{2} = \pi \alpha c \, \frac{Z_1 Z_2}{v}$$

- The convolution with the velocity distribution

$$P(v; v + \mathrm{d}v) = f(v)\, \mathrm{d}v \propto v^2 e^{-mv^2/2kT}\, \mathrm{d}v$$

gives the reaction rate of the plasma

$$\mathrm{R} = \int N\, Nv\, \sigma(v)\, T\, f(v)\, \mathrm{d}v$$

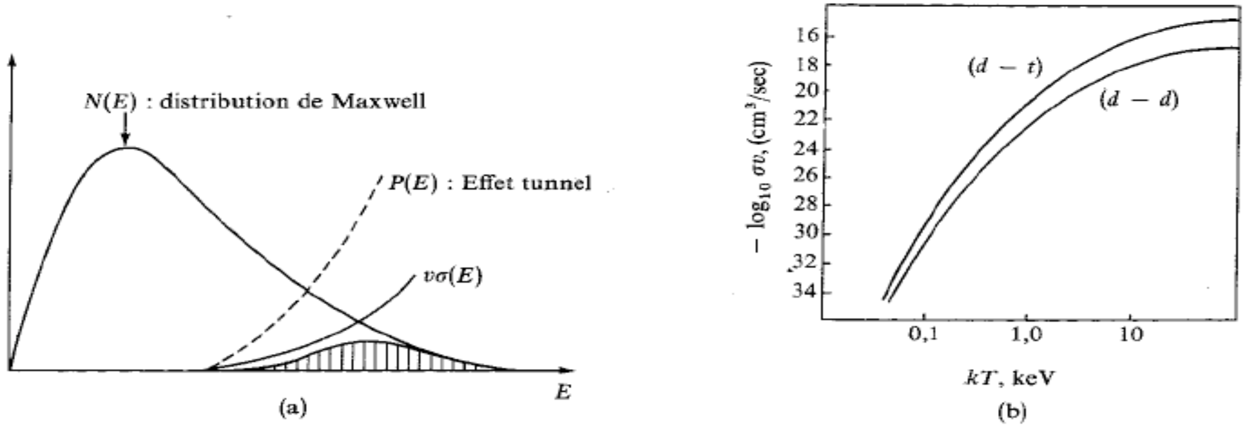
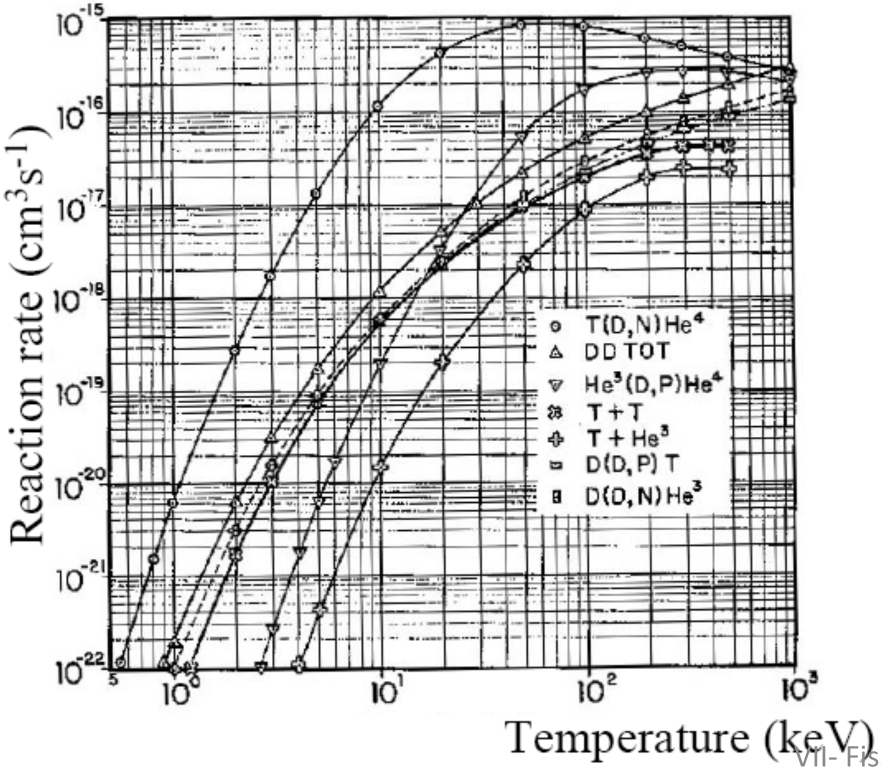


FIGURE VI.3

La zone hachurée sur la figure (a) représente le résultat du calcul de $\langle \sigma v \rangle$. La figure (b) donne le logarithme à base 10 de cette valeur moyenne, dite taux de réaction par paire, en fonction de la température d'équilibre du plasma, pour les réactions (d-d) et (d-t).



- Typical fusion reactions peak at *kT*≈100 keV
  ⇒ $T=10^{9}$K
- Lets consider the reaction:

$$p + p \longrightarrow d + e^{+} + \nu_e$$

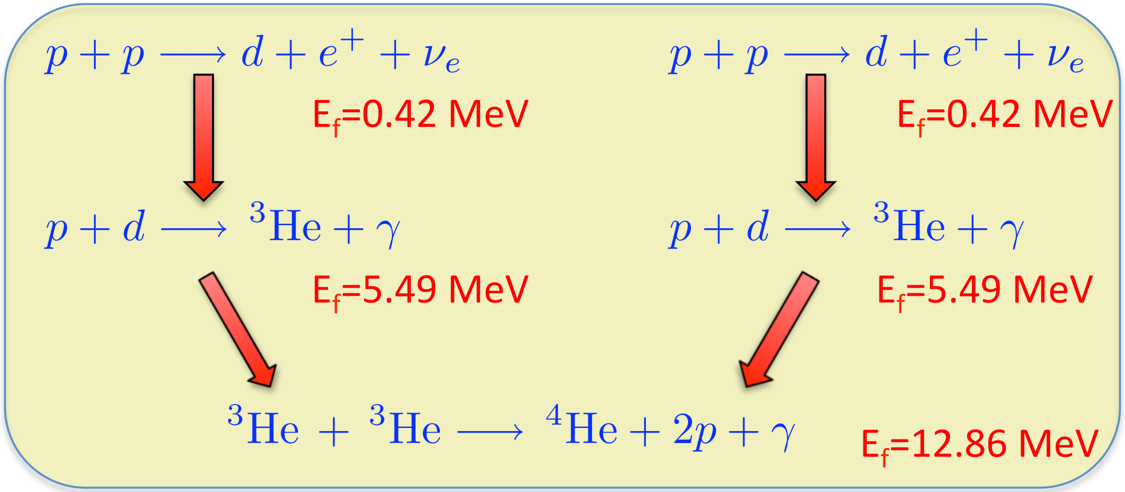
$\sigma \approx 10^{-32}$b → very small value, but compensated by the large number of protons in the sun.

Reaction Rate/proton/s ≈ $5\ 10^{-18}$ s

⇒At this rate , the sun mean life is 10^{10} years

Fusion processes in the sun

PPI Chain



$$p + p \longrightarrow d + e^+ + \nu_e \qquad p + p \longrightarrow d + e^+ + \nu_e$$

E_f=0.42 MeV $\qquad$ E_f=0.42 MeV

$$p + d \longrightarrow {}^3\mathrm{He} + \gamma \qquad p + d \longrightarrow {}^3\mathrm{He} + \gamma$$

E_f=5.49 MeV $\qquad$ E_f=5.49 MeV

$${}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2p + \gamma$$

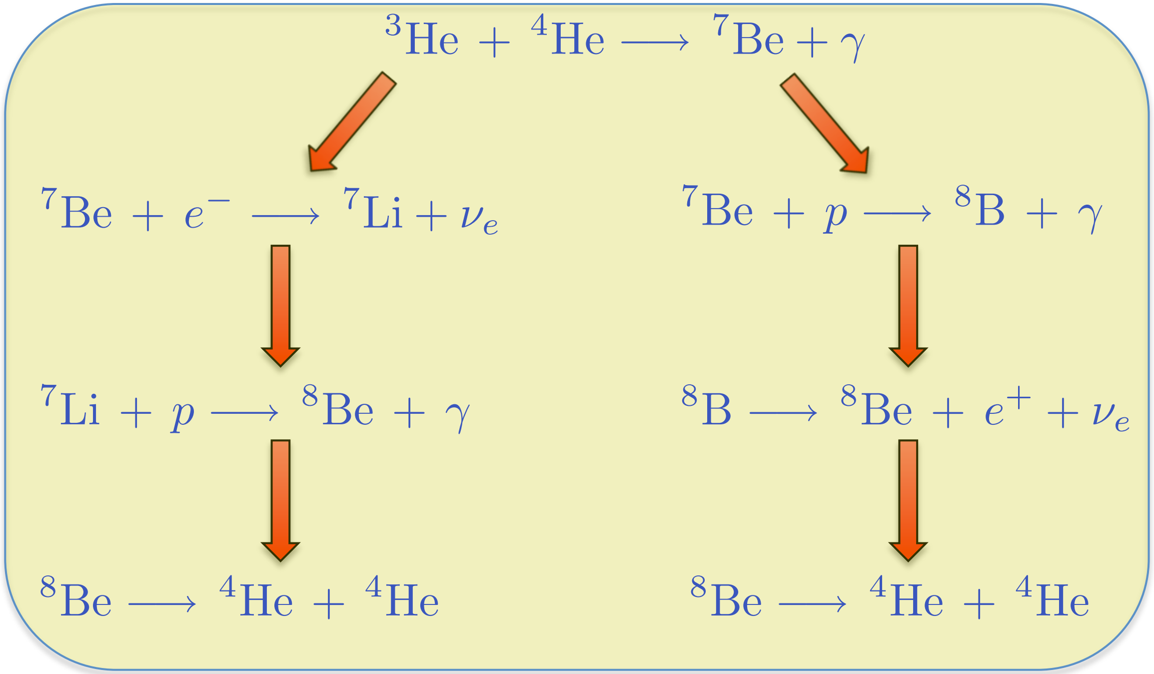
E_f=12.86 MeV

- Net reaction: $4\,p \longrightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e$

  $2e^+$ annihilate with $2e^-$ → E_{e+e-}=2.04 MeV
- Total energy release in the PPI cycle = 26.7 MeV

  (energy release per proton=26.7/4=6.7 MeV)
- Neutrinos emerge without further interactions with ~2% of the energy. The rest heats the core and is released as electromagnetic energy at the photosphere
- Observed luminosity : 4×10^{26} J/s → 4×10^{38} protons consumed each second

- Other He cycles (Helium acts as a catalyst) :



$$^{3}\mathrm{He} + {}^{4}\mathrm{He} \longrightarrow {}^{7}\mathrm{Be} + \gamma$$

$$^{7}\mathrm{Be} + e^{-} \longrightarrow {}^{7}\mathrm{Li} + \nu_e$$

$$^{7}\mathrm{Li} + p \longrightarrow {}^{8}\mathrm{Be} + \gamma$$

$$^{8}\mathrm{Be} \longrightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

$$^{7}\mathrm{Be} + p \longrightarrow {}^{8}\mathrm{B} + \gamma$$

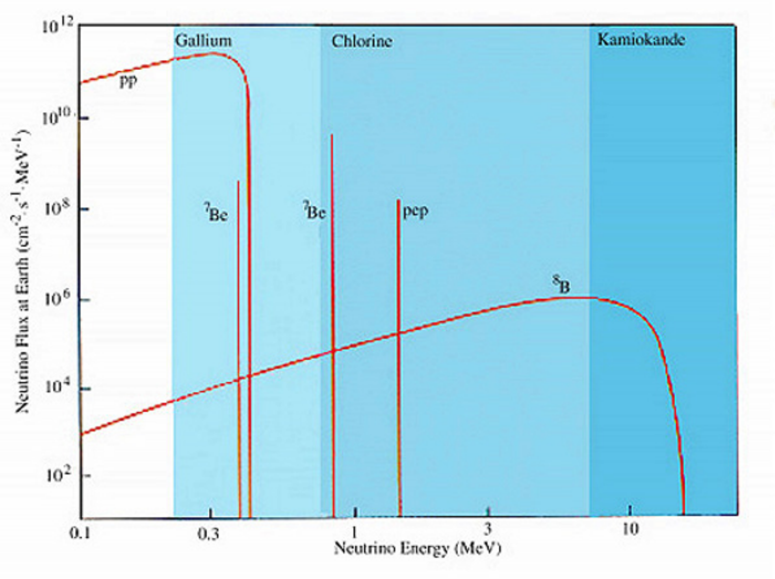
$$^{8}\mathrm{B} \longrightarrow {}^{8}\mathrm{Be} + e^{+} + \nu_e$$

$$^{8}\mathrm{Be} \longrightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

PPII Chain **PPIII Chain**

- For T > 3×10^{7}K, PPIII is dominant
  (p are able to cross the coulomb barrier of ^{7}Be faster than the rate of the CE in ^{7}Be)
- There are other cycles like CNO here C, N and O act as catalyst.
- Currently in the sun : PPI=56%, PPII=40%, PPIII=0.05%, CNO=3.2%

Solar neutrinos



- Observation of solar neutrinos from the various sources tests the stellar model. Probes directly the core of the sun where the thermonuclear reactions are taking place. Many experiments are measuring the solar neutrino flux

- Expected flux depends on
  - Standard solar model (temperature, density,…)
  - Nuclear reaction cross section
- Solar neutrino problem : observed flux = 1/3 expected flux
- This problem has recently been resolved by seeing a non-ν_e component in the nuclear flux
  ⇒Neutrino oscillations (see particle physics course)

2.3 Further fusion process in stars

- Hydrogen combustion : principal sequence stars, 10^{10} years, T≈2 10^{7} K
- When the hydrogen is exhausted, further gravitational collapse occurs and the temperature rises. Eventually the ^{4}He starts to burn via fusion : red giant stars, 10^{5} years, T≈2 10^{8} K

$$^4\mathrm{He} + {}^8\mathrm{Be} \longrightarrow {}^{12}\mathrm{C} + \gamma$$

$$^4\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^8\mathrm{Be} + \gamma$$

$$^4\mathrm{He} + {}^{12}\mathrm{C} \longrightarrow {}^{16}0 + \gamma$$

- When the ^{4}He is exhausted, further collapse and further fusion reactions : supergiant stars, T≈ 4 10^{9} K

  for example: $^{12}\mathrm{C} + {}^{12}\mathrm{C} \longrightarrow {}^{24}\mathrm{Mg} + \gamma$ $\quad$ $^{16}\mathrm{O} + {}^{16}\mathrm{O} \longrightarrow {}^{28}\mathrm{Si} + \gamma$

  ⇒ends up near the most tightly bound nuclei with the doubly magic ^{56}Ni. This nucleus in β unstable and leads to ^{56}Fe on the stability line.

A word on nucleosynthesis

- Heavier elements than these are formed in supernova explosions, through reactions like the following:

$$n + {}^{56}\mathrm{Fe} \longrightarrow {}^{57}\mathrm{Fe} + \gamma$$

$$n + {}^{57}\mathrm{Fe} \longrightarrow {}^{58}\mathrm{Fe} + \gamma$$

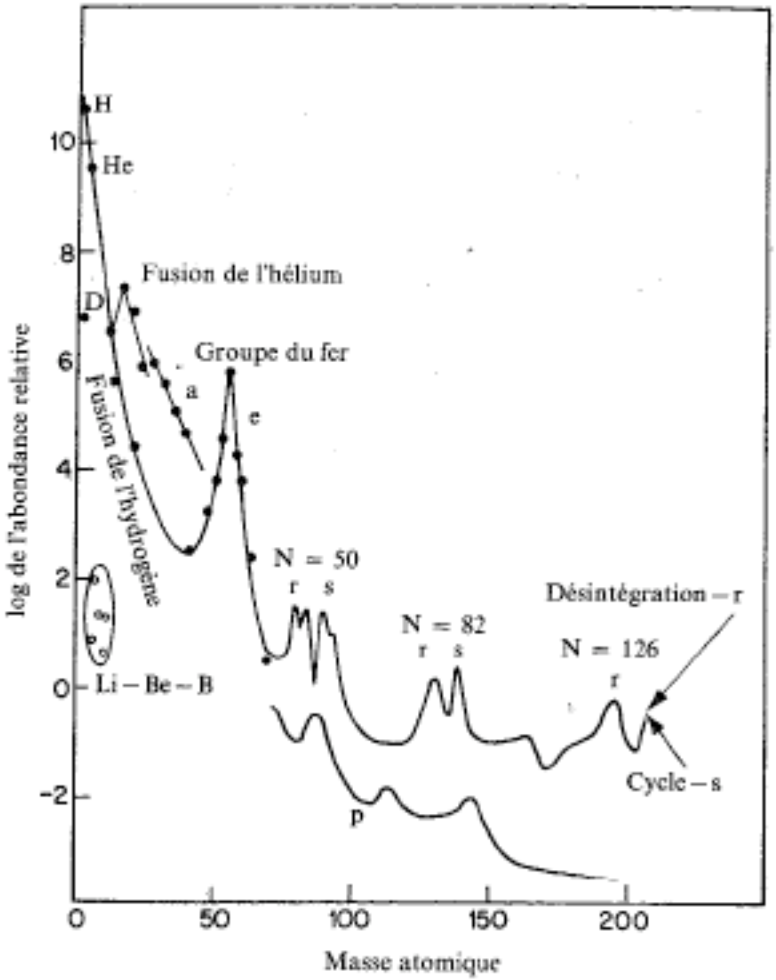
$$n + {}^{58}\mathrm{Fe} \longrightarrow {}^{59}\mathrm{Fe} + \gamma$$

$${}^{59}\mathrm{Fe} \longrightarrow {}^{59}\mathrm{Co} + e^- + \overline{\nu}_e$$

- These processes are based on neutron radiative capture (no coulomb barrier):

$$(A,Z)+n \rightarrow (A+1,Z)+\gamma$$

  - If (A+1,Z) is stable, it will undergo a new neutron capture, and so on…
  - This operation occurs x times, as long as the lifetime of the final isotope (A+x,Z) is long compared to the neutron capture rate. This rate depends heavily on the neutron flux.
- For a small neutron flux, the β decay is dominant and the nucleus (A+x,Z) will decay to (A+x,Z+1), and then capturing a neutron. This is the s process (s for slow). This process essentially occurs in equilibrium stars, and follows the stability line.
- For high neutron flux, the neutron capture is dominant until a very short lifetime isotope is reached. This is the r process (r for rapid). High neutron flux are produced during the explosive phases of the stars (like supernovae). This process explores the neutron rich side of the stability line



log de l'abondance relative
10
8
6
4
2
0
-2
H
He
D
Fusion de l'hélium
Fusion de l'hydrogène
Groupe du fer
a
e
Li – Be – B
N = 50
r
s
N = 82
r
s
N = 126
r
Désintégration – r
Cycle – s
p
0
50
100
150
200
Masse atomique

2.4- Controlled fusion

- The challenge is to reach sufficiently high temperatures to achieve fusion, under controlled conditions, with a gain in energy.
- Possible reactions (d=^{2}H=deuton, t=^{3}H=tritium):

$$d + d \longrightarrow {}^3\mathrm{He} + n \quad Q=3.3\ \mathrm{MeV}$$

$$d + d \longrightarrow t + p \quad Q=4.0\ \mathrm{MeV}$$

$$d + t \longrightarrow {}^4\mathrm{He} + n \quad Q=17.6\ \mathrm{MeV}$$

- The *d+t* reaction is especially attractive
  - ✓ Largest energy release (α is very stable)
  - ✓ Lowest coulomb barrier
- tritium is unstable ($T_{1/2}$=12 years). Can be produced via $n + {}^6\mathrm{Li} \longrightarrow \alpha + t$
- Need E≈10keV (i.e. T≈10^8K) to achieve a reasonable rate
→need to control a plasma
  - Magnetic confinement: use a configuration of magnetic field to control the plasma and keep it away from the walls
→ Tokamac: machine producing a toroidal magnetic field for confining a plasma
  - Plasma Heating: many different methods.
e.g: intertial confinement : pellet containing d+t zapped with lasers or particle beam to heat it.(→ laser Megajoule : 1.8 MJ laser)

ITER : first plasma in 2018
Goal : 500MW during 1000s