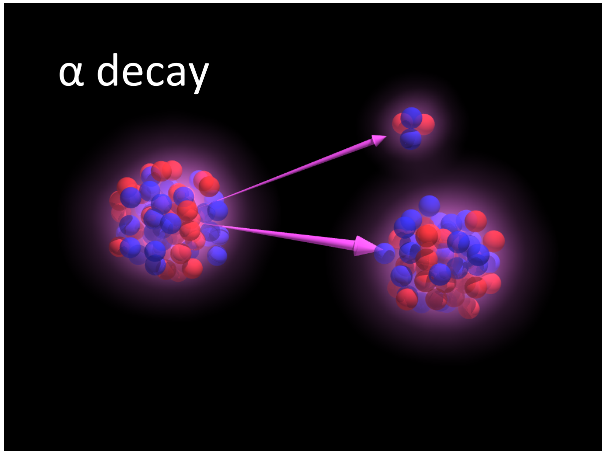
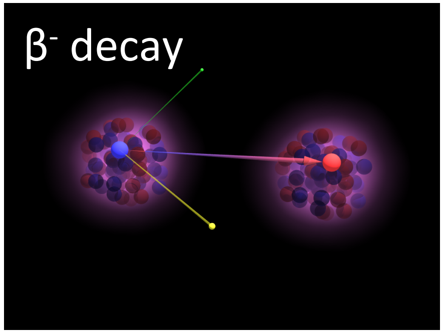
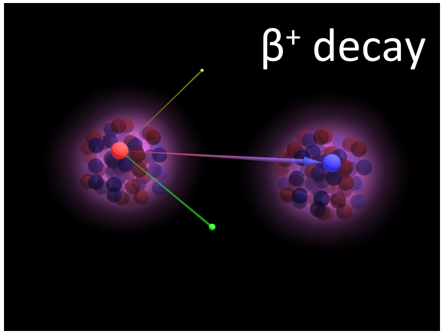
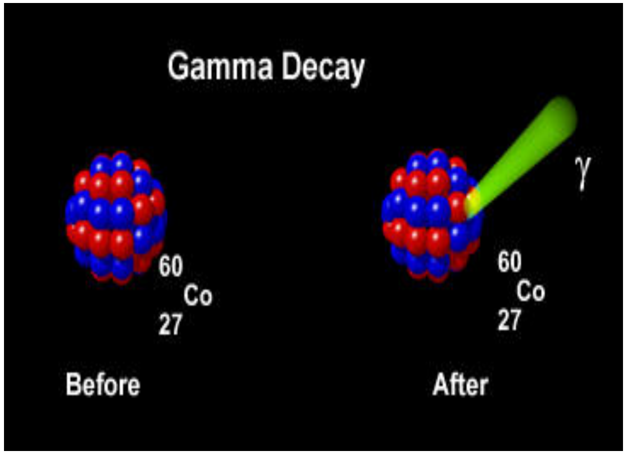
# Chapter 6
# α, β and γ decay



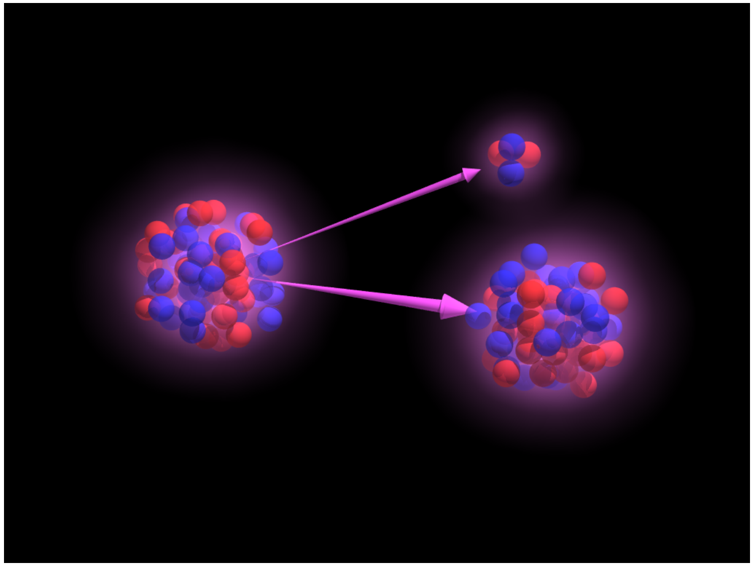



# Outline/Plan

1. α decay
   1. Experimental facts
   2. Gamow model
   3. Selection rules
2. β decay
   1. Experimental facts
   2. Fermi theory
   3. Selection rules
   4. Double β decay
3. γ decay

1. Désintégration α
   1. Faits expérimentaux
   2. Modèle de Gamow
   3. Règles de sélection
2. Désintégration β
   1. Faits expérimentaux
   2. Théorie de Fermi
   3. Règles de sélection
   4. Double désintégration β
3. Désintégration γ

# 1- α decay



$$ {}^{A}_{Z}\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He} $$

$$Q_\alpha = M(X) - M(Y) - M(^4He)$$

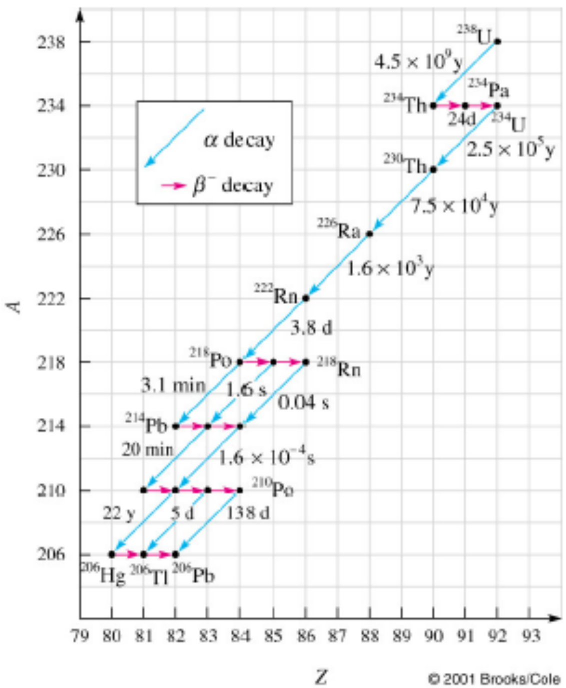
## Reminder:

- α-decay is due to the emission of a $^4$He nucleus
- $^4$He is a doubly magic nuclei and is very tightly bound
  $B_\alpha \approx 28$ MeV
- α decay is energetically favorable for almost all nuclei having A≥190 and for many A≥150.

# 1.1- Experimental facts

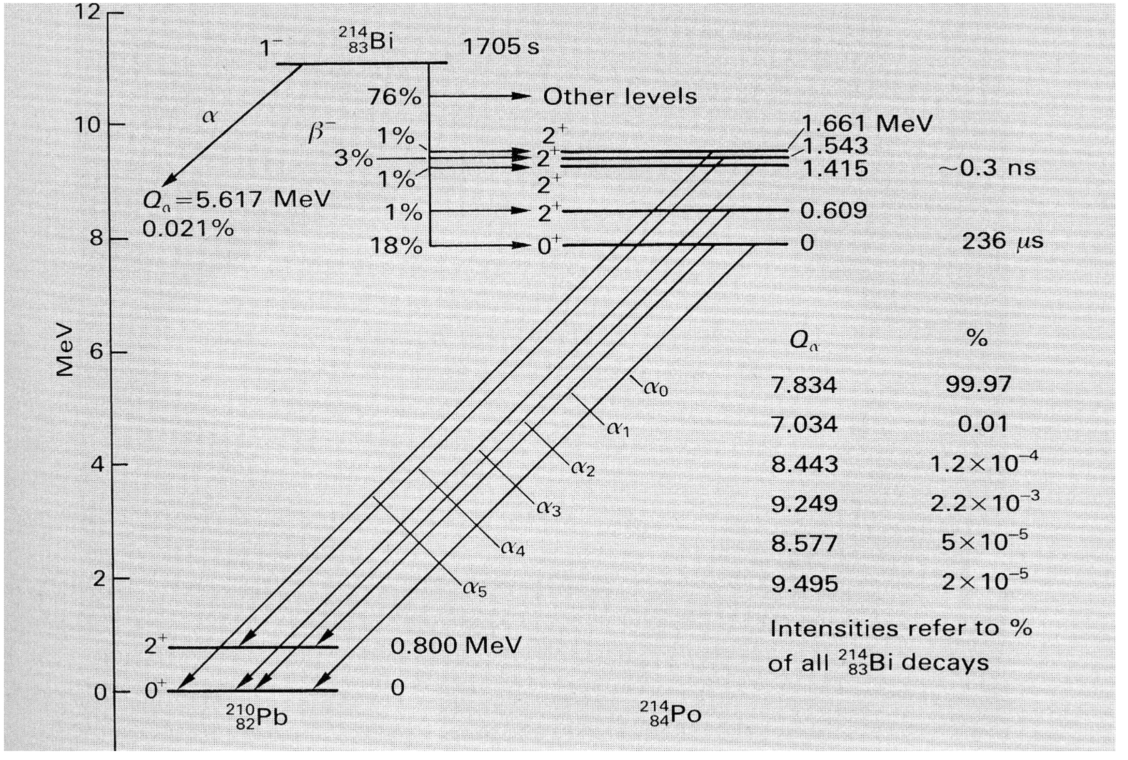
- Q-values of various possible decay of $^{232}$U

| | n | p | d | t | $^3$He | α | $^5$He | $^6$Li | $_7$Li |
|---|---|---|---|---|---|---|---|---|---|
| Q (MeV) | -7.26 | -6.12 | -10.70 | -10.24 | -9.92 | +5.41 | -2.59 | -3.79 | -1.94 |




Natural radioactivity

- Typical α-decay spectrum ($^{214}Po \rightarrow ^{210}Pb$)




- Discrete energy spectrum

# Energy release in α-decay for various A>200 nuclei

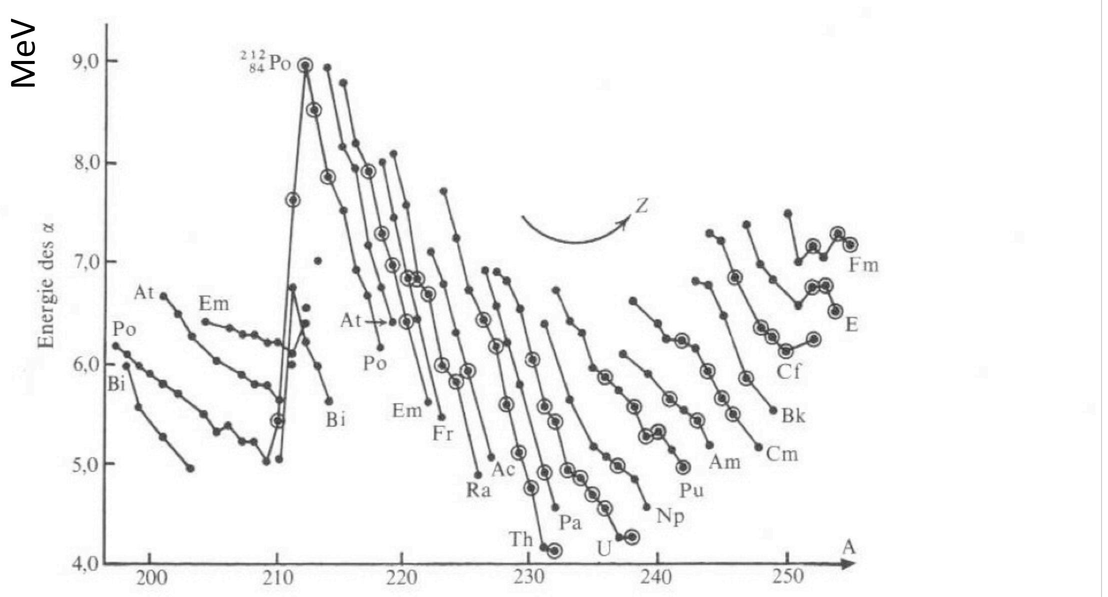


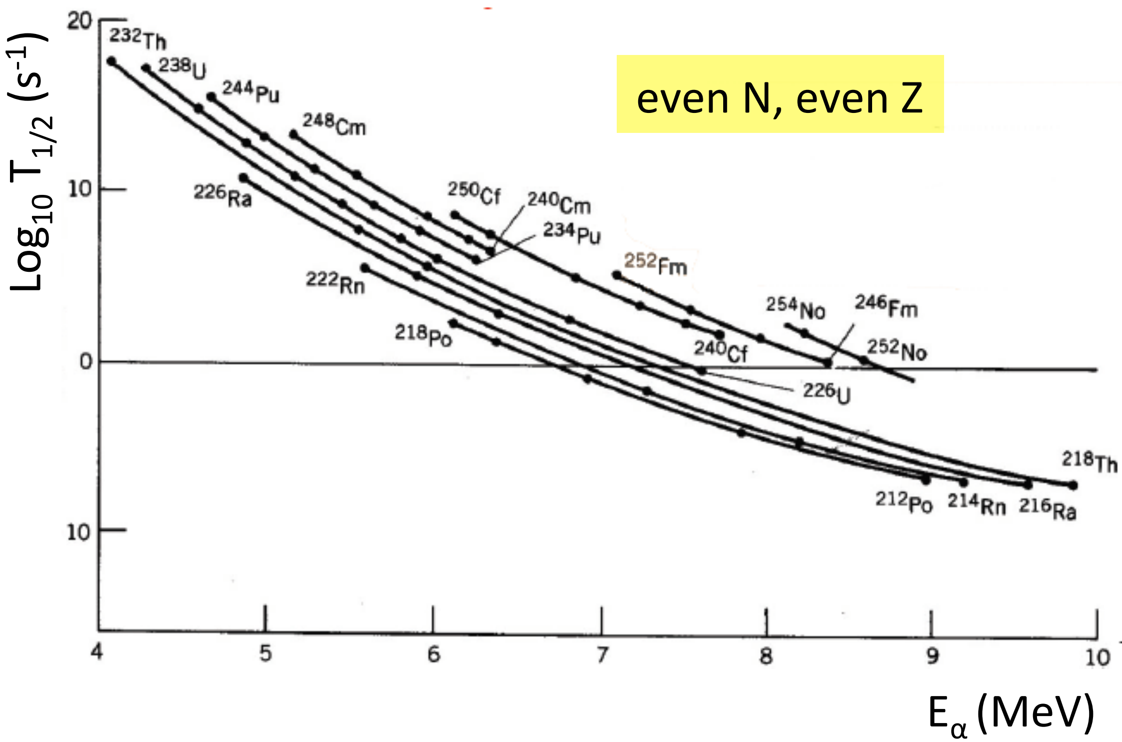

FIGURE V.7

**Le changement brutal observé autour de $A = 212$ constitue une mise en évidence supplémentaire des effets de couches nucléaires. Le noyau $^{208}$Pb est doublement magique ($N = 126$, $Z = 82$). Les lignes joignent les isotopes.**

General trend :

- in an isotopic series, $E_\alpha$ decreases with increasing A (some local accident due to the shell effect, see the polonium series)
- For a given A, $E_\alpha$ increases with Z
- $E_\alpha$ lie roughly between 5 and 10 MeV

# Dependence of $T_{1/2}$ with respect to $E_\alpha$



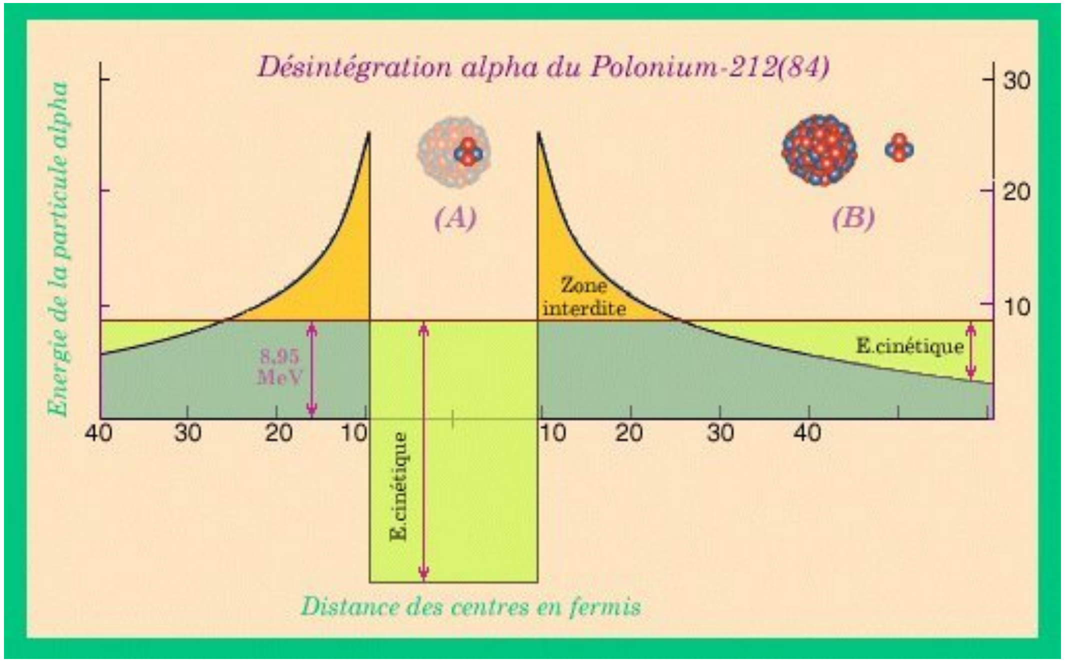

- Geiger and Nuttal 1911
- Very strong dependence of lifetime on Eα
- For a given isotopic chain, $T_{1/2}$ falls with A

| | $E_\alpha$ | $T_{1/2}$ |
|---|---|---|
| $^{232}$Th | 4.08 MeV | 1.4 $10^{10}$ years |
| $^{218}$Th | 9.85 MeV | 1.0 $10^{-7}$ s |

**⇒ factor 2.5 in $E_\alpha$ leads to a factor $10^{24}$ in $T_{1/2}$**

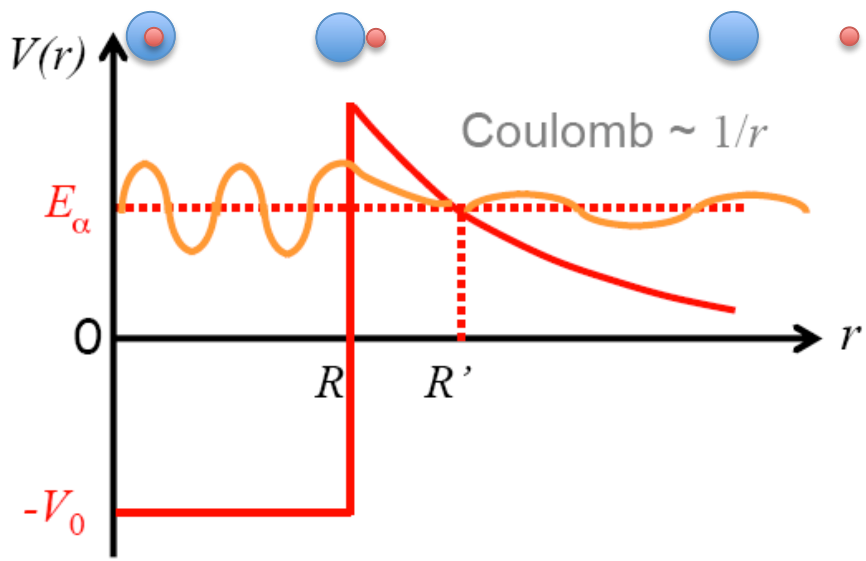
# 1.2 Gamov model

Based on the crossing of a coulomb barrier via quantum mechanical tunneling.




- Classically, the α particle cannot enter or escape the nuclei
- Quantum mechanically, the α particle can penetrate the Coulomb barrier.

The nuclear potential for the α particle due to the daughter nucleus includes a coulomb barrier which inhibits the decay.




- The nuclear potential is described by a square well whose depth is $V_0$. The width of the well is R corresponding to the sum a the radii of the daughter nuclei and the α particle
- The coulomb potential between the α-particle (charge 2) and the daughter nuclei (charge Z'=Z-2) is:

$$V(r) = \frac{2Z'\alpha\hbar c}{r} = \frac{k}{r}$$

- $E_\alpha$ is the kinetic energy of the α-particle in the final state.
- R', is the distance where the α-particle escapes the coulomb barrier:

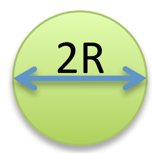
$$V(R') = E_\alpha \Rightarrow R' = \frac{Z'\alpha\hbar c}{E_\alpha}$$

Gamov model (1928): the α-decay probability λ can be expressed by:

$$\lambda_\alpha = p(\alpha)\, f\, T$$

✓ $p(\alpha)$ is the probability of existence of a bound α particle in the nucleus. The extent to which α-particles really exist inside nucleus still debatable. In this model, we assume that α exists : $p(\alpha) = 1$

✓ $f$ is the escape trial frequency, i.e. the collision frequency between the α and the barrier. Classically:




$$f = v/2R$$

where $v$ is the velocity of the α-particle inside the nucleus: $v = \sqrt{2E_0/m}$

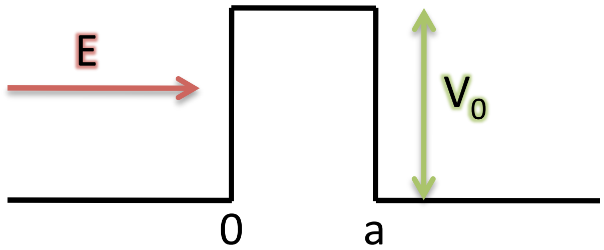
$$f = \frac{1}{R}\sqrt{\frac{E_0}{2m}}$$

for $V_0$=45MeV, $E_\alpha$≈5 MeV → $E_0$ = 50 MeV (α energy inside the nucleus): $f \approx 10^{21}\ s^{-1}$

$m_\alpha$≈3.7 GeV; R ≈ 7 fm (A=200)

✓ $T$ is the tunneling probability

- Standard case of a 1-d square barrier




The tunneling probability T, is obtained by solving the Schrödinger equation in the three regions and using boundary and continuity conditions on the wavefunctions.

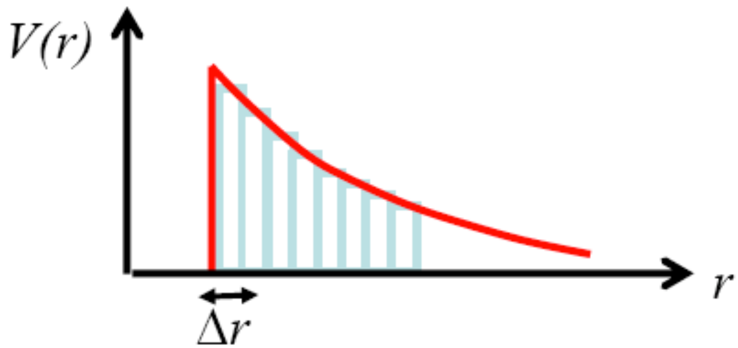
$$T = \left[1 + \frac{V_0^2}{4(V_0 - E)E\sinh^2 ka}\right]^{-1}$$

with $\hbar k = \sqrt{2m(V_0 - E)}$

in the limit $k\,a \gg 1$, $T$ is dominated by exponential decay in the barrier :

$$T \simeq e^{-2ka}$$

- For a coulomb potential, $V \propto 1/r$. Divide into small rectangular pieces and multiply the crossing probabilities, i.e. multiply together exponentials, i.e. sum exponents




k depends on r

- The probability to tunnel trough the coulomb barrier is

$$T = \prod_i e^{-2k_i \Delta r} = e^{-2\int_R^{R'} k(r)\mathrm{d}r} \quad \text{with} \quad k(r) = \frac{\sqrt{2m(V(r) - E_\alpha)}}{\hbar}$$

- Let's define G, the Gamow factor

$$G = \int_R^{R'} k(r)\,\mathrm{d}r = \int_R^{R'} \frac{\sqrt{2m(V(r) - E_\alpha)}}{\hbar}\,\mathrm{d}r$$

- The probability is then: $T = e^{-2G}$

- For r > R, $V(r) = \frac{2Z'\alpha\hbar c}{r} = \frac{\kappa}{r}$ with $Z' = Z_{daugther} = Z - 2$

$\alpha$ particle escapes coulomb barrier at $r = R'$, $V(R') = E_\alpha \Longrightarrow R' = \kappa/E_\alpha$

$$G = \left(\frac{2m}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[\frac{\kappa}{r} - E_\alpha\right]^{1/2} \mathrm{d}r = \left(\frac{2m\kappa}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[\frac{1}{r} - \frac{1}{R'}\right]^{1/2} \mathrm{d}r$$

after integration (detailed calculation in TD)

$$G = \sqrt{\frac{2m}{E_\alpha}}\frac{\kappa}{\hbar}\left[\arccos\sqrt{\frac{R}{R'}} - 2\sqrt{\left(1 - \frac{R}{R'}\right)\frac{R}{R'}}\right]$$

- In most of the practical cases, R’≫R, the term [...] can be approached by π/2

$$G = \sqrt{\frac{2m}{E_\alpha}}\frac{\kappa}{\hbar}\frac{\pi}{2}$$

$$G = \sqrt{2mc^2}\,\pi\alpha\frac{Z'}{\sqrt{E_\alpha}}$$

- Example : Z=90, $E_\alpha$=7 MeV ⇒ R' ≈ 50 fm and R ≈ 6 fm ⇒ R' ≫ R

$$G \simeq 1.97\,\frac{Z'}{\sqrt{E_\alpha}}$$

- Putting everything back together:

$$\lambda_\alpha = p(\alpha)\,f\,T = \frac{v}{2R}\,e^{-2G} \qquad\qquad \ln\lambda_\alpha = -2G + \ln\left(v/2R\right)$$

$$\lambda_\alpha = \sqrt{\frac{E_\alpha}{2m_\alpha c^2}}\,\frac{c}{r_0 A^{1/3}} \times e^{-\pi\alpha\sqrt{8m_\alpha c^2}\frac{Z'}{\sqrt{E_\alpha}}} = a\,e^{-b\frac{Z'}{\sqrt{E_\alpha}}}$$

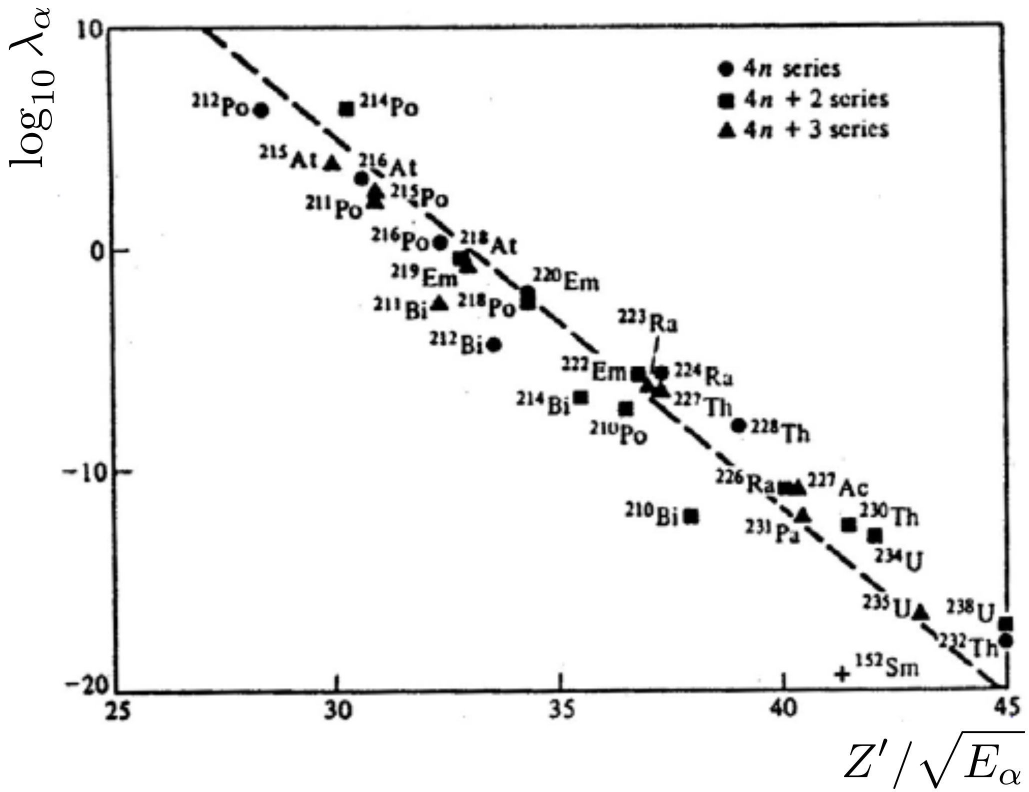
$$\ln\lambda_\alpha = -b\frac{Z'}{\sqrt{E_\alpha}} + \ln a$$

**Geiger-Nuttal Law**

$$\ln\lambda_\alpha \propto -\frac{Z'}{\sqrt{E_\alpha}} + \text{constant}$$

$$\ln T_{1/2} \propto \frac{Z'}{\sqrt{E_\alpha}} + \text{constant}$$

# Geiger Nuttal Law




$$\ln\lambda_\alpha \propto -\frac{Z'}{\sqrt{E_\alpha}} + \text{constant}$$

Good explanation of the trend of the data (i.e. short lived isotopes emit more energetic alpha particles than long lived ones)

Simple tunneling model explains:

✓ the strong dependence between the lifetime and energy of the α particle

✓ the increase of lifetime with Z

Since $G \propto m^{1/2}$ and $G \propto Z_{fragment}$ , it also explains why the decay to heavier fragments like $^{12}C$ is disfavored

# Oversimplifications / ameliorations

- We have assumed the existence of a bound α in the nucleus. We should have taken into account its probability of formation
- We have followed a classical approach in the determination of the escape trial frequency.
- Rigorously the α mass appearing in the model, should be the reduced mass of the system (for heavy nucleus very small changes)
- If the α particle is emitted with a non zero angular momentum, $\ell$, the Schrödinger radial equation includes a centrifugal barrier term:

$$V_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}$$

where $\ell$ is the relative angular momentum of the α particle and the daughter nucleus, and $\mu$ is the reduced mass.
High $\ell$ values raise the barrier → Emission of high $\ell$-states α particles are disfavored (this is observed)

# 1.3- Selection rules in α decay

- Angular momentum

$$\underset{J_X}{X} \longrightarrow \underbrace{\underset{J_Y}{Y} + \underset{0}{\alpha}}_{\ell}$$

  - α is even-even : $J^\pi=0^+$
  - Conservation of total angular momentum: $\vec{J}_X = \vec{J}_Y + \vec{\ell}$

$$|\vec{J}_X - \vec{J}_Y| \leq \ell \leq \vec{J}_X + \vec{J}_Y$$

- Parity
  - Conserved in α decay
  - Parity of orbital wavefunction: $(-1)^\ell$

If X and Y have the same parity → $\ell$ must be even

If X and Y have an opposite parity → $\ell$ must be odd

✓ Between the ground states of even-even nuclei: $\ell=0$

✓ If X has 0+, reachable states of Y : $J^\pi=0^+,1^-,2^+,3^-,\ldots$

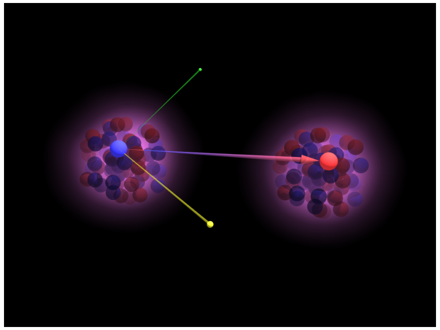
# 2- β decay

- β⁻ decay $\,{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + e^{-} + \bar{\nu}_e$

  at nucleon level $n \to p + e^{-} + \overline{\nu}_e$

  energy condition

  $m(X) > m(Y) + m_e + m_\nu \Longrightarrow M(X) > M(Y) + m_\nu$



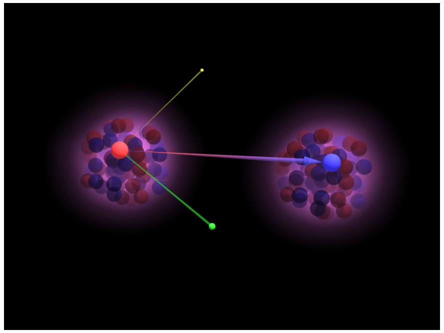
- β⁺ decay $\,{}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + e^{+} + \nu_e$

  $n \to p + e^{-} + \overline{\nu}_e$

  at nucleon level $p \to n + e^{+} + \nu_e$

  energy condition

  $m(X) > m(Y) + m_e + m_\nu \Longrightarrow M(X) > M(Y) + 2m_e + m_\nu$

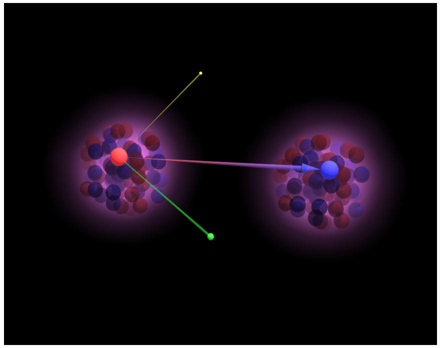


- Electron capture $\,e^{-} + {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + \nu_e$

  at nucleon level $p + e^{-} \to n + \nu_e$

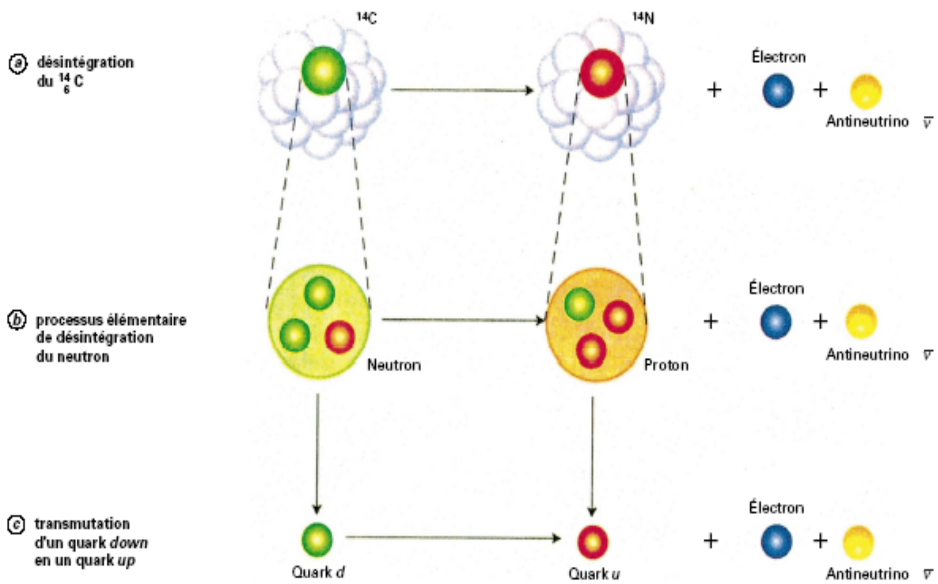
  energy condition

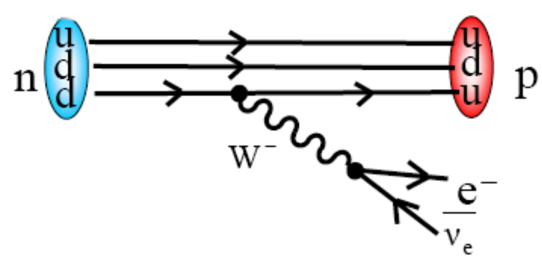
  $m(X) > m(Y) - m_e + m_\nu + B_K \Longrightarrow M(X) > M(Y) + B_K$



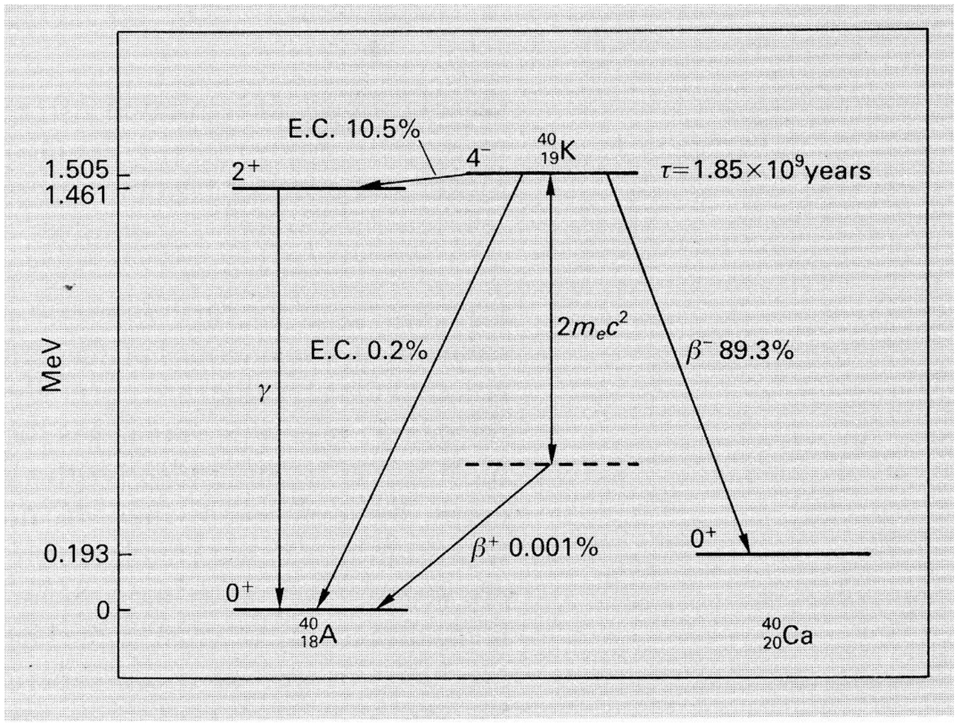
# 2.1 Experimental facts

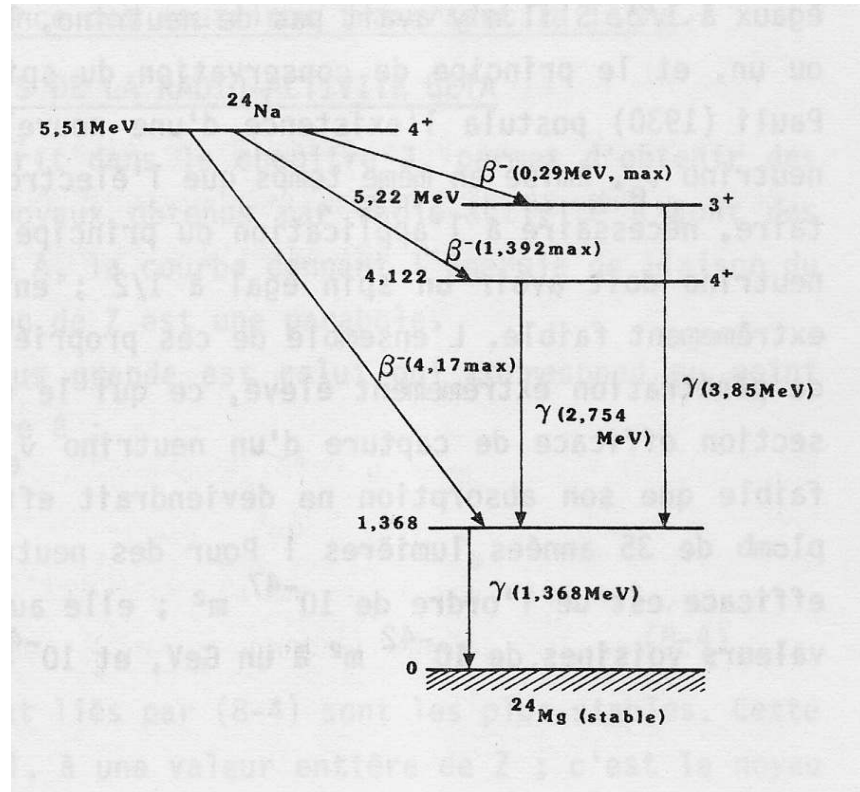
- β decay occurs via weak interaction, and is mediated by the W boson ($m_W$=80 GeV/$c^2$)



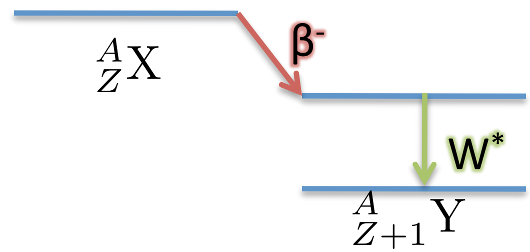
# Example of β decay



# Energy conservation : example of β⁻ decay

$$ {}^A_Z\mathrm{X} \longrightarrow {}^{A}_{Z+1}\,\mathrm{Y} + e^- + \bar{\nu}_e $$



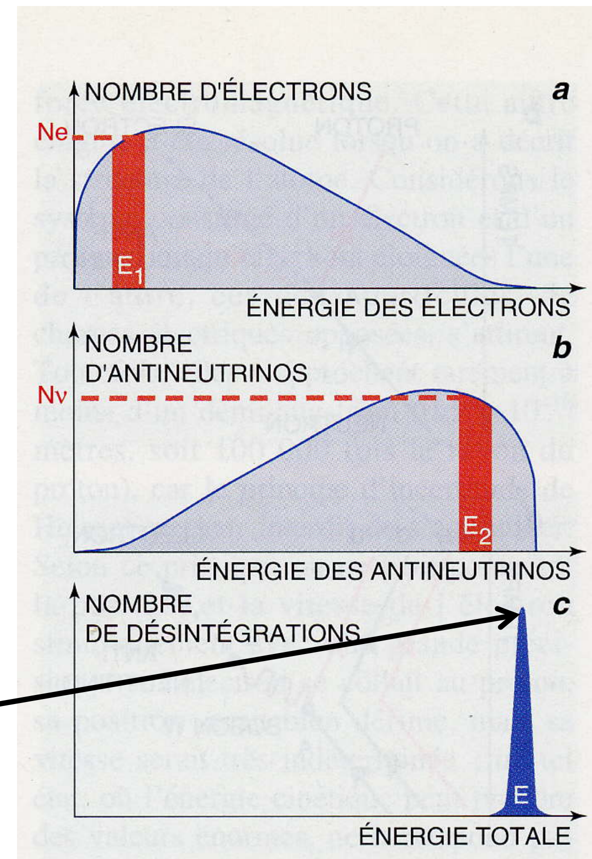

- Conservation of energy: $m(X) = m(Y) + m_e + m_\nu + K_Y + K_e + K_\nu + W^*$
  - Neutrino mass : $m_{\nu e} < 2$ eV
    (source Particle Data Group, 2008, http://pdg.lbl.gov/)
  - For simplification, assume $W^*=0$
  - Nuclear recoil negligible : classic two body decay
    (assume ν+e = β, one particle)

$$ \begin{cases} K_Y = p_Y^2/2m_Y \\ K_\beta = p_\beta^2/2m_\beta \end{cases} $$

$$ \Longrightarrow K_Y = \frac{m_\beta}{m_Y} K_\beta \Longrightarrow K_Y \ll K_\beta $$

  - Let's define

$$ K_\beta^{\max} = K_e + K_\nu = m(X) - m(Y) - m_e $$



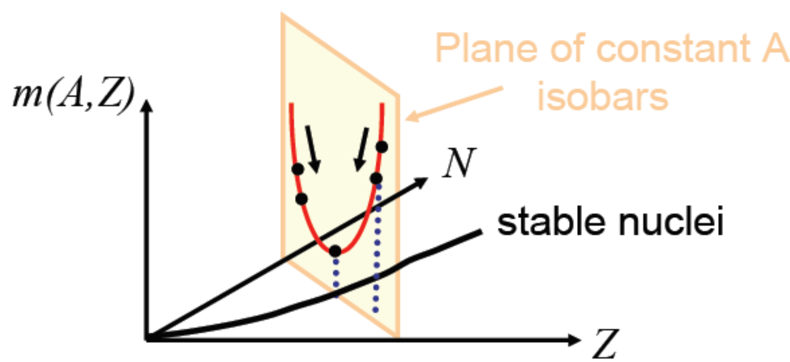
# β stability

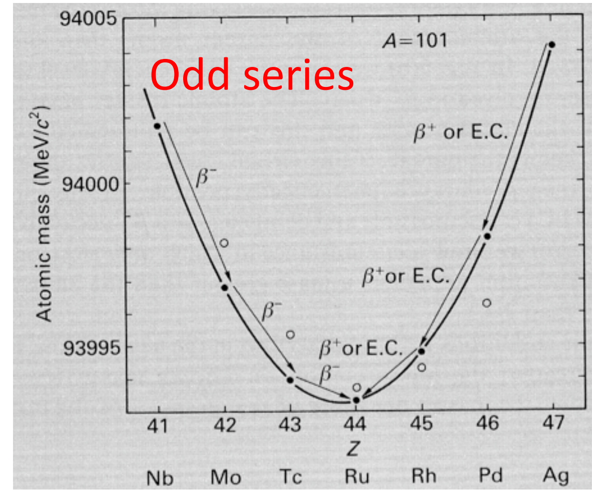
$$M(A,Z) = ZM(^1H) + (A-Z)m_n - u_v A + u_s A^{2/3} + u_c \frac{Z^2}{A^{1/3}} + u_T \frac{(A-2Z)^2}{A} - \delta(A)$$

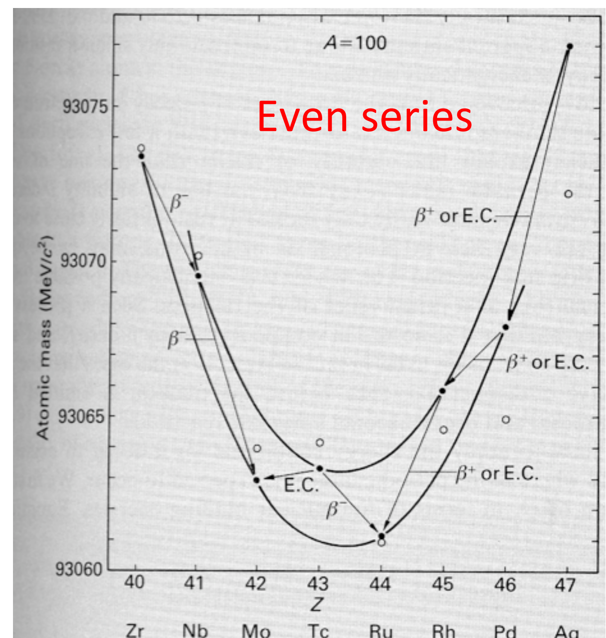
cf lecture 4 p26-29

β decay:

- Isobaric (A constant)
- Z changes by ±1
- M(A,Z) is quadratic in Z (min for $Z_0$) → mass parabola



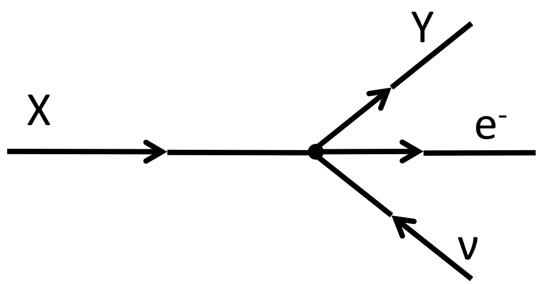



# 2.2- Fermi theory of β decay

**(In this section, we use natural units ħ=c=1)**

- Weak interaction describer as contact interaction involving 4 fermions




Simplified approach: in reality depends also on spins and momenta

  - No propagator. Absorb the effect of the exchanged W boson into an effective coupling strength given by the Fermi constant $\mathbf{G_F}$
  - $G_F$=1.166 $10^{-5}$ $\mathrm{GeV}^{-2}$
- Use Fermi's Golden Rule to obtain the transition rate

$$\Gamma = 2\pi |M_{fi}|^2 \rho(E_f)$$

where $M_{fi}$ is the matrix element of the perturbing hamiltonian
(perturbative treatment justified since weak interaction is really weak)

and $\rho(E_f) = \dfrac{\mathrm{d}N}{\mathrm{d}E_f}$ is the density of final states

# ➢ Density of final states

- Three body final state: consider two particles for the density of states, the third one is given by energy-momentum conservation. Choose $e^-, \nu_e$

$$\mathrm{d}^6 N = \frac{p_\nu^2\,\mathrm{d}p_\nu \mathrm{d}\Omega_\nu}{(2\pi)^3}\,\frac{p_e^2\,\mathrm{d}p_e \mathrm{d}\Omega_e}{(2\pi)^3}$$

- The energy $E_0$ released in β decay is shared between the electron $E_e$, the neutrino $E_\nu$ and the recoiled kinetic energy $T_Y$ of the daughter nucleus
(we have previously seen that since the nucleus is much more massive, the recoil energy is negligible)

$$E_0 \simeq E_e + E_\nu$$

for the neutrino $E_\nu = p_\nu$
for a given electron energy $E_e$: $\mathrm{d}p_\nu = \mathrm{d}E_\nu = \mathrm{d}E_0$

$$\frac{\mathrm{d}^6 N}{\mathrm{d}E_0} = \frac{\mathrm{d}^6 N}{\mathrm{d}p_\nu} = \frac{E_\nu^2\, p_e^2}{(2\pi)^6}\,\mathrm{d}\Omega_\nu\,\mathrm{d}\Omega_e\,\mathrm{d}p_e$$

assuming an isotropic decay, the integration over the angular variables gives:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}E_0} = \frac{E_\nu^2\, p_e^2}{4\pi^4}\,\mathrm{d}p_e = \frac{(E_0 - E_e)^2\, p_e^2}{4\pi^4}\,\mathrm{d}p_e = \mathrm{d}\rho(E_F)$$

- Using the Fermi's Golden Rule with this "differential" state density

$$\mathrm{d}\Gamma = 2\pi |M_{fi}|^2 \, \mathrm{d}\rho(E_f)$$

$$\mathrm{d}\Gamma = 2\pi |M_{fi}|^2 \, \frac{(E_0 - E_e)^2 \, p_e^2}{4\pi^4} \, \mathrm{d}p_e$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}p_e} = |M_{fi}|^2 \, \frac{(E_0 - E_e)^2 \, p_e^2}{2\pi^3}$$

## ➢ Matrix Element

using the Fermi 4 points contact interaction: $M_{fi} = G_F \int \psi_p^* \, \psi_e^* \, \psi_\nu^* \, \psi_n \, \mathrm{d}^3\vec{r}$

After the decay, e and ν are free particles:

$\psi_e = e^{i\,\vec{p}_e.\vec{r}}$ and $\psi_\nu = e^{i\,\vec{p}_\nu.\vec{r}}$

$$M_{fi} = G_F \int \psi_p^* \, \psi_n \, e^{-i(\vec{p}_e + \vec{p}_\nu).\vec{r}} \, \mathrm{d}^3\vec{r} = G_F \, M_{\mathrm{nucl}}$$

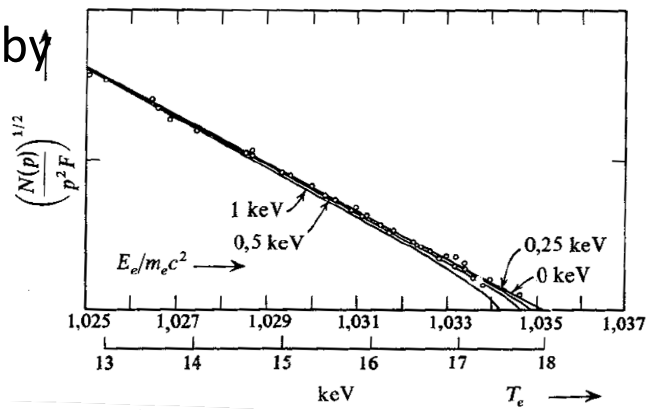
Putting everything together:

$$d\Gamma = \frac{G_F^2}{2\pi^3} |M_{\text{nucl}}|^2 \, (E_0 - E_e)^2 \, p_e^2 \, dp_e$$

Integrating over $p_e$ gives the total decay rate::

$$\Gamma = \frac{G_F^2}{2\pi^3} |M_{\text{nucl}}|^2 \int_0^{E_0} (E_0 - E_e)^2 \, p_e^2 \, dp_e$$

- Electron momentum decay spectrum is described by




$$\sqrt{\frac{d\Gamma}{dp_e} \frac{1}{p_e^2}} \propto (E_0 - E_e)$$ **Kurie line**

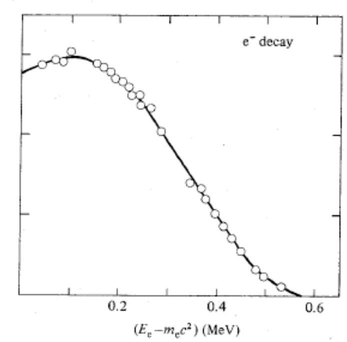
- In the relativist limit, $p_e \approx E_e$. The total decay rate is given by:

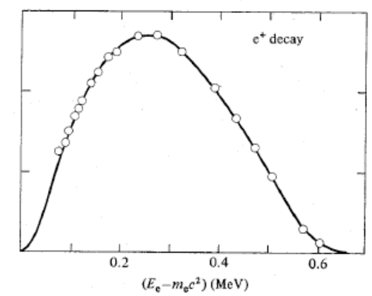
$$\Gamma \propto E_0^5$$ **Sargent's rule**

# Improvement:

- The momentum of the electron is modified by the Coulomb interaction with the nucleus (differently for $e^-$ and $e^+$)
  → Effect embedded in the Fermi function $F(Z_Y, p_e)$

$$\Longrightarrow \Gamma = \frac{G_F^2}{2\pi^3} |M_{\rm nucl}|^2 \int_0^{E_0} (E_0 - E_e)^2 \, p_e^2 \, F(Z_Y, p_e) \, {\rm d}p_e$$




- Values of the complicated Fermi Integral $f(Z_Y, p_e)$

$$f(Z_Y, E_0) = \frac{1}{m_e^5} \int_0^{E_0} (E_0 - E_e)^2 \, p_e^2 \, F(Z_Y, p_e) \, {\rm d}p_e$$

  are tabulated

- From the half-life, $T_{1/2} = \ln 2/\Gamma$, it is possible to form the product $fT_{1/2}$ which depends only on the nuclear matrix element

$$fT_{1/2} = \ln 2 \, \frac{2\pi^3}{m_e^5 \, G_F^2 \, |M_{\rm nucl}|^2}$$

Classification of β decay according to $fT_{1/2}$

| $\log_{10}(fT_{1/2})$ | denomination |
|---|---|
| 3-4 | superallowed |
| 4-7 | allowed |
| >6 | forbidden |

Suppressed, i.e. small M →

Non examinable

# 2.3- Selection Rules in β decay

(essentially descriptive)

- **Allowed transitions**
  relative angular momentum $\ell$ of of the (e,ν) pair to the nucleus, $\ell$=0

$$e^{-i(\vec{p}_e+\vec{p}_\nu).\vec{r}} \simeq 1 \qquad \log fT_{1/2} \simeq 4-7$$

- **Superallowed transitions** (subset of allowed transitions, often between mirror nuclei in which p and n have the same wavefunction)

$$M_{\mathrm{nucl}} = \int \psi_p^* \, \psi_n \, \mathrm{d}^3\vec{r} \simeq 1 \qquad \log fT_{1/2} \simeq 3-4$$

- e,ν : fermions spin ½ → the total spin of the (e,ν) pair can be $S_\beta$=0 or 1
  → there are two type of transitions depending on the relative spin states of (e,ν)
  - $S_\beta$=0: Fermi transitions ΔJ=0 →

$$n\uparrow \to p\uparrow + \tfrac{1}{\sqrt{2}}\underbrace{\left[e^-\uparrow \bar{\nu}_e\downarrow - e^-\downarrow \bar{\nu}_e\uparrow\right]}_{S_{e\nu}=0} \qquad J_X = J_Y$$

  - $S_\beta$=1: Gamow-Teller transitions ΔJ=0,±1 →

$$n\uparrow \to p\downarrow + \underbrace{e^-\uparrow + \bar{\nu}_e\uparrow}_{S_{e\nu}=\pm 1} \qquad J_X = J_Y$$

$$n\uparrow \to p\uparrow + \tfrac{1}{\sqrt{2}}\underbrace{\left[e^-\uparrow \bar{\nu}_e\downarrow + e^-\downarrow \bar{\nu}_e\uparrow\right]}_{S_{e\nu}=1,\ S_z=0}$$

  - Since $\ell$=0, parity of the nucleus unchanged

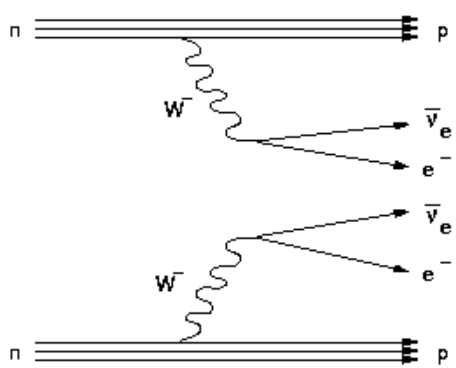
- **Forbidden transitions** $\log fT_{1/2} > 6$
  - relative angular momentum of of the (e,ν) pair to the nucleus $\ell>0$
  - Transition probabilities for large $\ell$ are small ( $\vec{p}.\vec{r} \simeq 10^{-3}$ ) → transitions highly suppressed
  - Forbidden transitions are only competitive if an allowed transitions can not occur (because of the selection rules). In general, the lowest permitted order of "forbiddeness" will dominate
  - In general, $n^{th}$ forbidden →(e,ν) pair carries orbital angular momentum $\ell=n$, and $S_\beta=0$ (Fermi) or $S_\beta=1$ (GT)
- **Some examples**

| Mother | daughter | Classif. | type | $\ell$ | $S_\beta$ | $T_{1/2}$ | $K_{max}$ |
|---|---|---|---|---|---|---|---|
| $^{34}$Cl ($0^+$) | $^{34}$S ($0^+$) | Allowed | Fermi | 0 | 0 | 1.52 s | 4.47 MeV |
| $^{14}$C ($0^+$) | $^{14}$N ($1^+$) | Allowed | GT | 0 | 1 | 5730 y | 156 keV |
| $^{39}$Ar ($7/2^-$) | $^{39}$K($3/2^+$) | $1^{st}$ forbidden | GT | 1 | 1 | 269 y | 565 keV |
| $^{87}$Rb ($3/2^-$) | $^{87}$Sr ($9/2^+$) | $3^{rd}$ forbidden | Fermi/GT | 3 | 0 or 1 | 4.75 $10^{10}$ y | 283 keV |

# 2.4 Double β decay

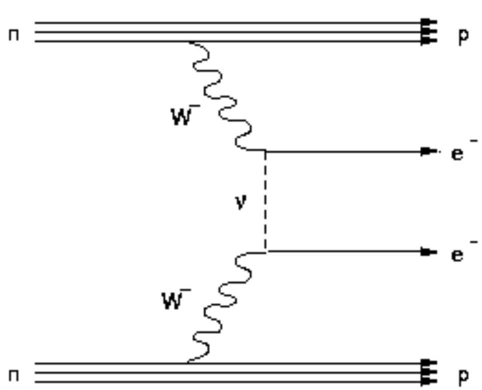
## ➢ Two neutrinos double β decay

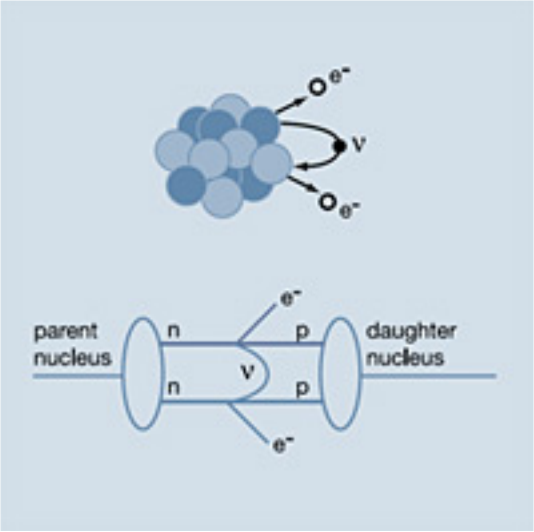
- In double-$\beta^-$ decay, two neutrons in the nucleus are converted to protons, and two electrons and two electron anti-neutrinos are emitted
  $^{82}Se \rightarrow {}^{82}Kr + 2e^- + 2\,\bar{\nu}_e$   $T_{1/2}=10^{20}$ yr   (observed in 1986)
- Double-β decay is the rarest known kind of radioactive decay; it was observed for only 10 isotopes, and all of them have a mean life time of more than $10^{19}$ yr.
- The conversion of two protons to neutrons, with emission of two electron neutrinos and absorption of two orbital electrons (double electron capture), of if the atom mass difference is greater than $4m_e$ with the emission of two positrons is theoretically possible, but these double-beta decay branches have not yet been observed.



# ➢ Neutrinoless double β decay

- If the anti-neutrino and the neutrino are actually the same particle (in this case the neutrino has to be massive), then it is possible for neutrinoless double-beta decay to occur.
- The two neutrinos annihilate each other, or equivalently, one nucleon absorbs the neutrino emitted by another nucleon of the nucleus
- To a very good approximation, the two electrons are emitted back-to-back to conserve momentum
- Numerous experiments have been carried out to search for neutrinoless double-beta decay. No success for the moment



# 3- γ decay

(very descriptive)

- Emission of γ-rays (electromagnetic radiation) occurs when a nucleus is created in an excited state (e.g. following α, β decay or collision)

$$ {}^{A}_{Z}\mathrm{X}^{*} \longrightarrow {}^{A}_{Z}\mathrm{X} + \gamma $$

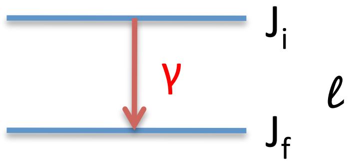
- The photons energies are discrete

- The treatment of radiative transitions in nuclei is essentially the same as for atoms. (involves the quantification of the electromagnetic field)

| | Eγ | wavelength | Transition rate | comment |
|---|---|---|---|---|
| Atom | ~ eV | $10^8$ fm<br>= 1000 × $r_{atom}$ | $10^9$ $s^{-1}$ | Only dipole transitions are important |
| Nuclei | ~ MeV | $10^2$ fm<br>= 25 × $r_{nucl}$ | $10^{16}$ $s^{-1}$ | Higher order transitions<br>also important |

- There are two types of transitions:
  - Electric (E) transitions: arise from an oscillating charge which causes an oscillation in the external electric field
  - Magnetic (M) transitions: arise from a varying current or a magnetic moment which sets up a varying magnetic field
- The photon carries away net angular momentum $\ell$ (a mixture of orbital + spin) when a proton in the nucleus makes a transition from its initial state $J_i$ to final state $J_f$

$$\vec{J}_i = \vec{\ell} + \vec{J}_f \implies |J_i - J_f| \ll \ell \ll J_i + J_f$$




- Single γ emission is forbidden for a transition between two J=0 states.
  0→0 transitions can only occur via internal conversion (emitting an electron)
- $\ell$ Is called the multipolarity of the transition
- A transition is described by the nature (E or M) and the multipolarity of the transition: $E\ell$ or $M\ell$

- The transition probabilities may be obtained using Fermi's Golden Rule
  → beyond the scope of this course
- The perturbing Hamiltonian is a function of electric and magnetic fields and hence of the electromagnetic 4-vector potential

$$M_{fi} = \langle\psi_f|\, H'(\vec{A})\, |\psi_i\rangle$$

- For a photon, A has the form of a plane wave and can be expanded in multipoles

$$e^{-i\vec{k}.\vec{r}} = 1 - i\vec{k}.\vec{r} + \frac{1}{2}(\vec{k}.\vec{r})^2 - ... + \frac{(-i\vec{k}.\vec{r})^n}{n!}$$

dipole $\ell$=1 quadrupole $\ell$=2 octupole $\ell$=3

- Each successive term in A is reduced from the previous from a factor kR
  ex: E=p=k=1 MeV, R=5 fm → kR = 5 Mev.fm ≈ 0.0025 → $|kR|^2 \approx 10^{-3}$

$$\frac{\Gamma(E2)}{\Gamma(E1)} \simeq \frac{\Gamma(M2)}{\Gamma(M1)} \simeq 10^{-3}$$

- Electric dipole transition E1 (w/o demonstation)
  - Fermi Golden Rule :

$$\Gamma = \frac{\omega^3}{3\pi\epsilon_0 c^3 \hbar} |\langle \psi_f | e\vec{r} | \psi_i \rangle|^2$$

  - Crude estimate : $|\langle \psi_f | e\vec{r} | \psi_i \rangle|^2 \simeq |eR|^2 \implies \Gamma = \frac{4\alpha}{3} E_\gamma^3 R^2$
  - Example : $E_\gamma$=1 MeV, R=5 fm
    ⇒$\Gamma(E1)$ = 0.24 $MeV^3.fm^2$ = 0.24/($\hbar^3 c^2$) = $10^{16}$ $s^{-1}$
- Magnetic dipole transition M1 (w/o demonstation)
  - Matrix element : $|\langle \psi_f | \mu\vec{\sigma} | \psi_i \rangle|^2$ with $\mu$ magnetic moment $\sigma$ pauli matrix
  - Crude estimate: $|\langle \psi_f | \mu\vec{\sigma} | \psi_i \rangle| \simeq e\hbar/2m_p = \mu_N$

$$\frac{\Gamma(M1)}{\Gamma(E1)} \simeq \left(\frac{\mu_N}{er_0}\right)^2 A^{-2/3} \simeq 10^{-3}$$

  - Magnetic moment is even under a parity transformation. No change in parity.

# γ decay : summary

Behavior under parity:

- $E\ell$ transitions: parity = $(-1)^{\ell}$
- $M\ell$ transitions: parity = $(-1)^{\ell+1}$

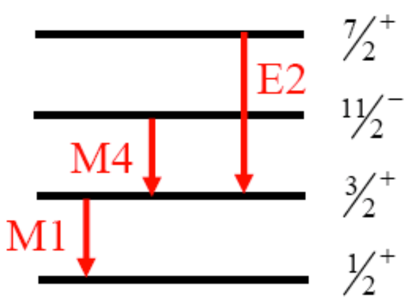
| Normalized Rate | 1 | $10^{-3}$ | $10^{-6}$ | $10^{-9}$ |
|---|---|---|---|---|
| Electric transitions | E1 | E2 | E3 | E4 |
| Magnetic transition | | M1 | M2 | M3 |
| Parity change | ✓ | ✗ | ✓ | ✗ |
| $J^{\pi}$ of **electric** γ | $1^-$ | $2^+$ | $3^-$ | $4^+$ |
| $J^{\pi}$ of **magnetic** γ | | $1^+$ | $2^-$ | $3^+$ |

- A decay will be dominated by the lowest order (i.e. fastest) process permitted by angular momentum and parity conservation.
  Often two multipolarities often dominate.

- The transition rate is also dependent on the energy of the photon and the number of nucleon A (volume of the nucleus)
  - $\Gamma(E\ell) \propto E_\gamma^{2\ell+1} A^{2\ell/3}$
  - $\Gamma(M\ell) \propto E_\gamma^{2\ell+1} A^{2(\ell-1)/3}$

  The relation between rates previously established can vary for large $E_\gamma$
- Experimentally, the information on the nature of the transitions is useful to determine the $J^\pi$ values of states.
- Keep in mind that the previous presentation is fairly naïve :
  - Collective effects may be important (more than one nucleon participate in transitions)
  - Deformation can enhance some transition rates.

Example: $^{117}_{50}\mathrm{Sn}$




$3/2^+ \rightarrow 1/2^+$ M1 (E2 also allowed)

$11/2^- \rightarrow 3/2^+$ M4 more likely than $11/2^- \rightarrow 1/2^+$ (E5)

$7/2^+ \rightarrow 3/2^+$ E2

M2, M3 } less likely: $7/2^+ \rightarrow 11/2^-$, $7/2^+ \rightarrow 1/2^+$