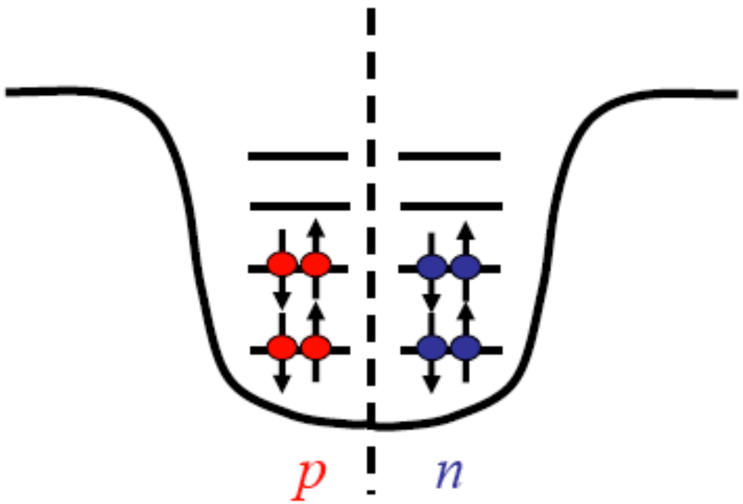
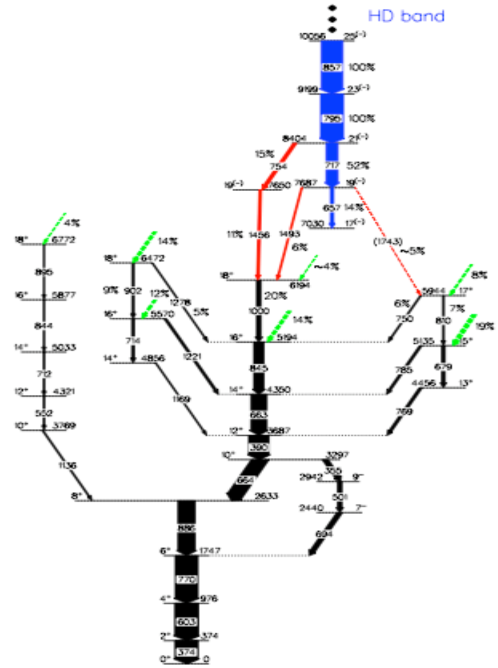
Chapter 5
Nuclear Structure





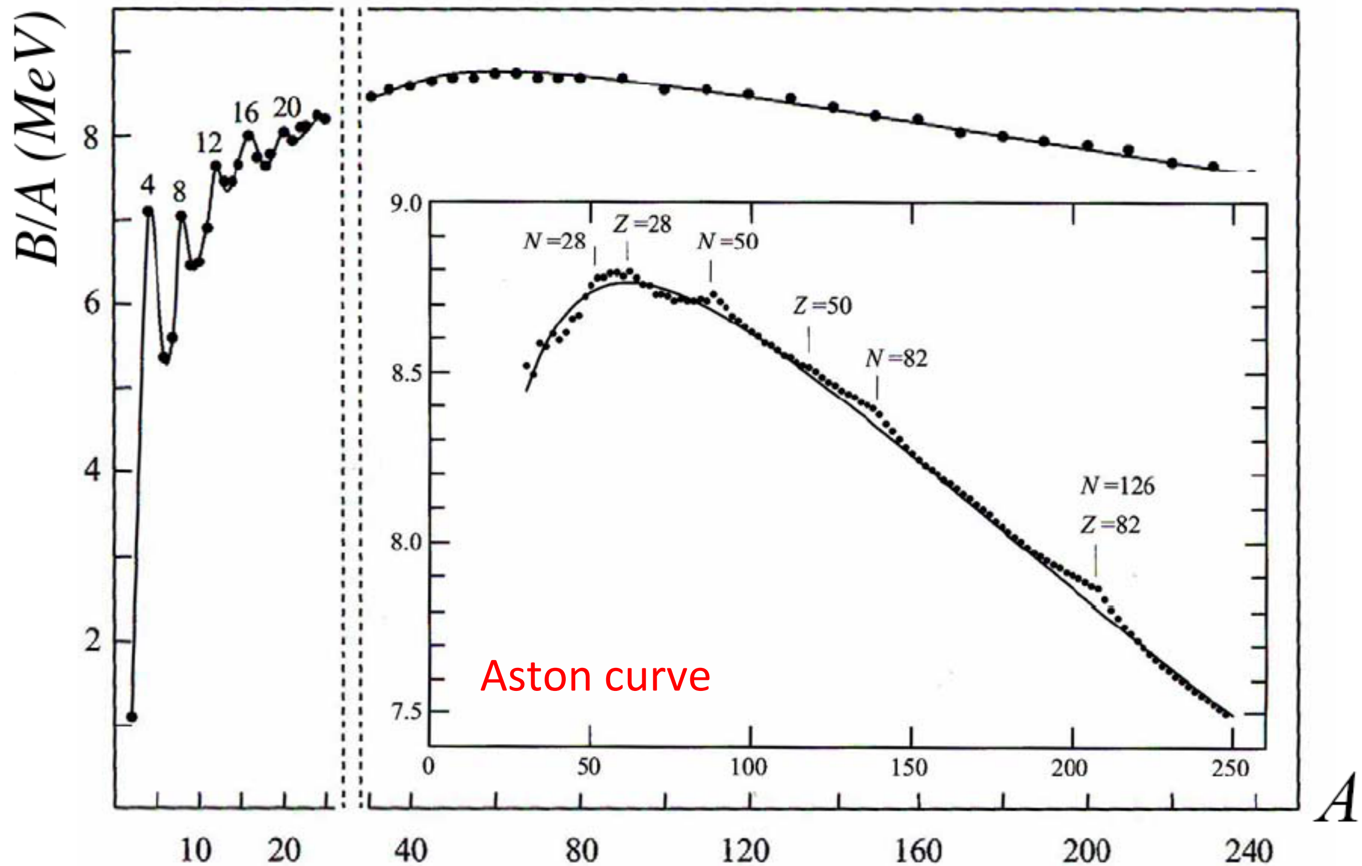
Outline/Plan

1- Experimental facts

Success and failure of the Semi-Empiric Mass Formula:

- Pretty good prediction of the nuclear mass, except for the light nuclei and some particular numbers of nucleons
- No explanations for the values of the parameters u_v, u_s, u_T and u_p
- Line of stability
- Stability versus βdecay
- Stability versus α decay
- Fission
- Excited states of the nuclei

1.1 Magic Numbers



For some values of Z or N, nuclei are very stable and show significant departures from the average nucleus behavior (LDM model):

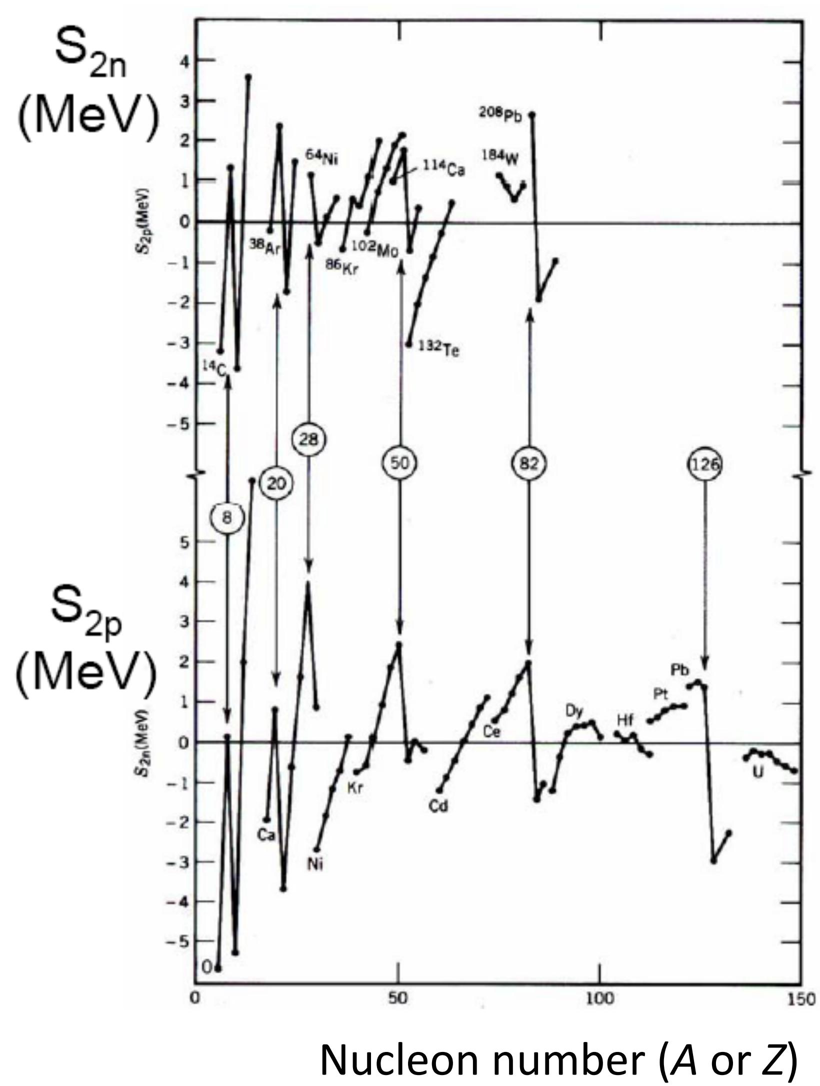
Z or N = 2, 8, 20, 28, 50, 82, 126 **"Magic numbers"**

Separation energy

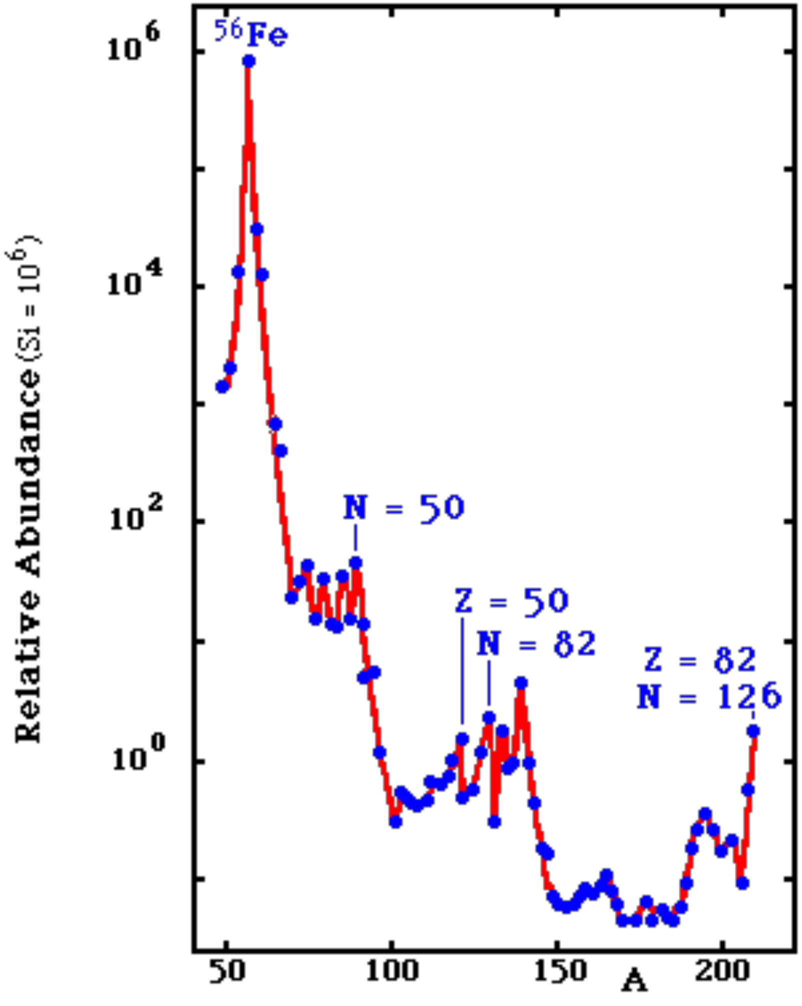
Other evidence for magic numbers:

Difference between experimental value and semi-empiric mass formula for the separation energy of 2 neutron (upper plot) and two proton (lower plot)

Again Z or N =2, 8, 20, 28, 50, 82, 126



Element Abundance



α energies

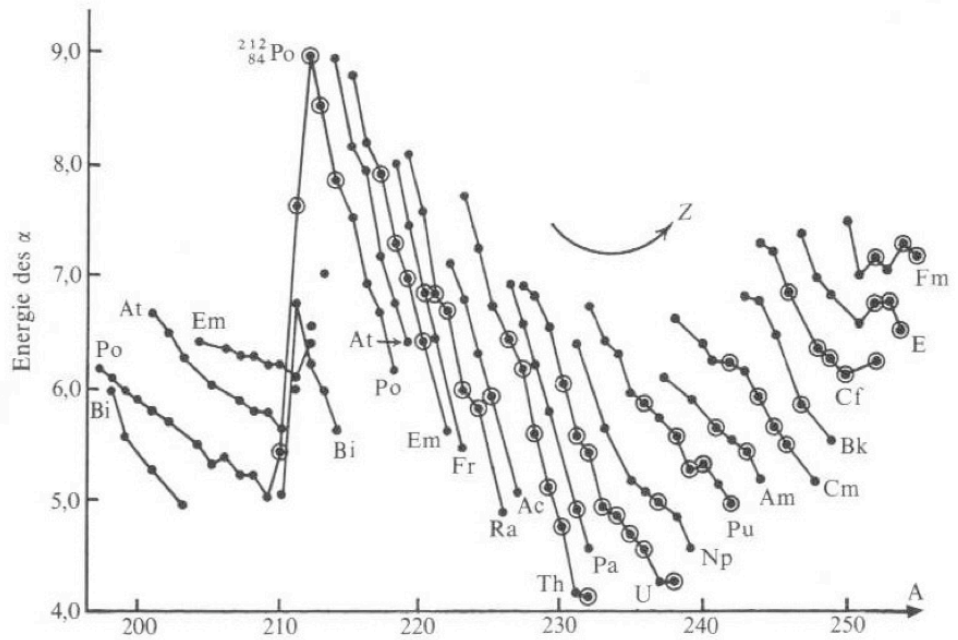


FIGURE V.7

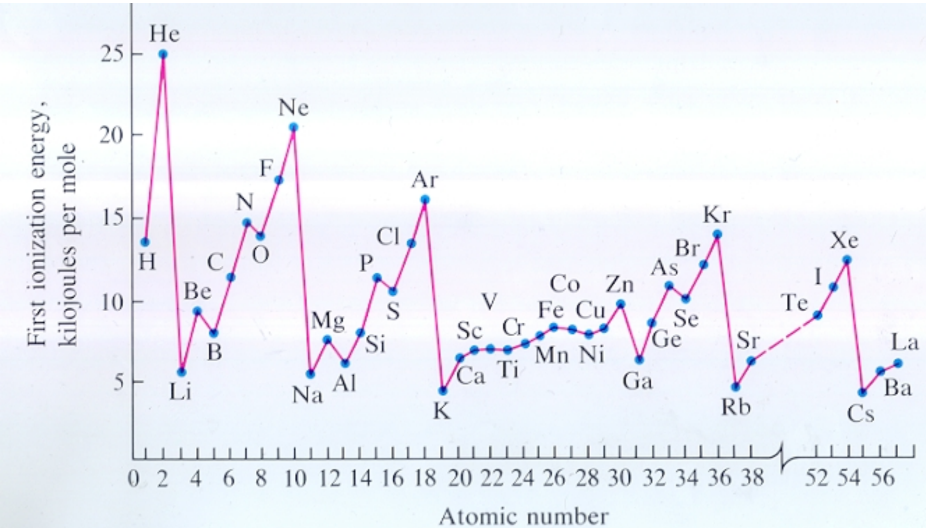
Le changement brutal observé autour de $A = 212$ constitue une mise en évidence supplémentaire des effets de couches nucléaires. Le noyau ^{208}Pb est doublement magique ($N = 126$, $Z = 82$). Les lignes joignent les isotopes.

Energy release in α, β decay is high when daughter nucleus is magic

Other evidence:

- Doubly magic nuclei are extremely stable:
  ^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, ^{48}Ni, ^{56}Ni, ^{100}Sn, ^{132}Sn and ^{208}Pb
  - They are among the most abundant (and stable) nuclei in the universe
  - ^{208}Pb is the heaviest stable nuclide
  - In 2006, hassium-270 (^{270}Hs, Z=108) was discovered having the unusually long half-life of 22 seconds. (suspected to be doubly-magic)
- Nuclear radius shows small change with *Z* or *N* at magic numbers.
- First excited states for magic numbers lie higher than neighbors
- Odd-A nuclei have small quadrupole moment when magic

1.2- Analogy with atomic physics



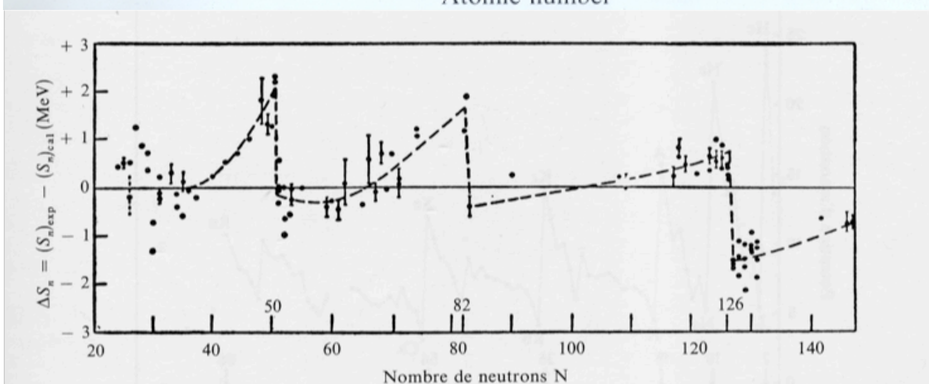


FIGURE V.5

L'écart entre S_{exp} et S_{cal} met en évidence des changements brutaux pour les nombres magiques 50, 82 et 126. Il existe des courbes semblables pour les protons.

- Similar behavior between the nucleon separation energy and the electron separation energy (ionization energy)
- Noble Gas Atoms ⇔ Magic Nuclei
- Shell structure ?

In the atomic case:

- Electrons move independently in a central potential
  $V(r)=k/r$ (coulomb field of the nucleus)
- Shells are filled progressively according to the Pauli exclusion principle.
- Properties of atom are defined by valence electrons
- Energy levels obtained by solving Schrödinger equation for a central potential

$$E_n \propto 1/n^2$$

2- Fermi gas model

Simple model useful to estimate the order of magnitude of the nuclear observables.

Model assumptions (also valid for shell model)

- Spherical nuclei (→ simplification of calculations)
  for practical purposes, magic nuclei have a spherical shape
- Static system
- Mean potential
  Nucleons move in a net nuclear potential that represents the average effect of two body interactions over the whole distribution of nuclear matter
- Independent particles

General idea: describe the nucleus as a degenerate fermion gaz.

- N fermions in a volume V in equilibrium at temperature T.
  F(E) is the probability that a state of energy E is filled:

$$F(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}}$$

- We will assume an extreme degeneracy, i.e. all low levels filled up to a maximum – the Fermi level (A gas is degenerate if E_F>>kT)
  We will assume that this is the case for a nucleus in its ground state.
- Nucleons in the nucleus are not relativist (E_{kin}<<M).
  We can define the Fermi momentum:

$$E_F = \frac{p_F^2}{2M}$$

Degenerate Fermion Gas

- Statistical physics *(Heisenberg uncertainty principle)*: the volume of an elementary cell in phase space is h^3
- In a space volume V, the number of states in a shell in p-space between p and $p+dp$ is:

$$\mathrm{d}n = \frac{V.4\pi p^2 \mathrm{d}p}{h^3}$$

- In the case of a fermion gas, the spin degeneracy is $g_s=2s+1=2$
- The number N of momentum states within the momentum-sphere up to p_F is:

$$N = \int_0^{p_F} g_s \, \mathrm{d}n = \frac{2V}{h^3} 4\pi \int_0^{p_F} p^2 \, \mathrm{d}p = \frac{8}{3} V \pi \, p_F^3$$

- The Fermi momentum is:

$$p_F = h \left(\frac{3N}{8\pi V} \right)^{1/3} = \hbar \left(\frac{3\pi^2 N}{V} \right)^{1/3}$$

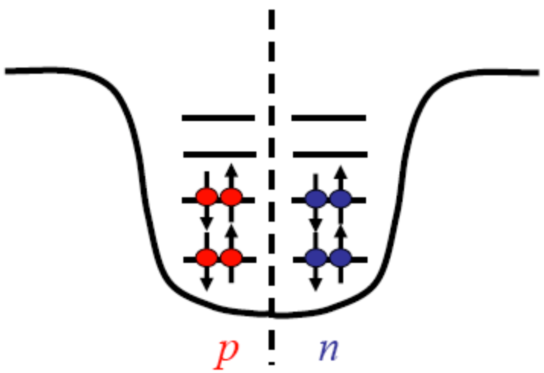
- The Fermi energy is:

$$E_F = \frac{p_F^2}{2M} = \frac{\hbar^2}{2M} \left(\frac{3\pi^2 N}{V} \right)^{2/3}$$

- The nucleus is made of two degenerate Fermi gas
  - a Z protons gas
  - a N neutrons gas

  enclosed in the same volume $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_0^3 A$

- The exclusion principle operates independently for protons and neutrons



- The proton and neutron Fermi energies are:

$$E_F(p) = \frac{\hbar^2}{2M_p}\left(\frac{3\pi^2 Z}{V}\right)^{2/3} \qquad E_F(n) = \frac{\hbar^2}{2M_n}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

- The total kinetic energy of the nucleons in the nuclear system is:

$$T_p(Z) = \sum_{\alpha=1}^{Z} T_\alpha = \int_0^{p_F(p)} \frac{p^2}{2M_p} \left[g_s \frac{V.4\pi p^2}{h^3} \right] \mathrm{d}p = \frac{3}{5} Z E_F(p)$$

$$T_n(N) = \sum_{\alpha=1}^{N} T_\alpha = \int_0^{p_F(n)} \frac{p^2}{2M} \left[g_s \frac{V.4\pi p^2}{h^3} \right] \mathrm{d}p = \frac{3}{5} N E_F(n)$$

$$T(A) = T_p(Z) + T_n(N) = \frac{3}{5}[N E_F(n) + Z E_F(p)]$$

- Assuming $M_p \approx M_n = M$ and $N \approx Z \approx A/2$

$$E_F(n) = E_F(p) = E_F = \frac{\hbar^2}{2Mr_0^2} \left(\frac{9\pi}{8} \right)^{2/3} \simeq 37MeV$$

$$T(A)/A \simeq \frac{3}{5} \frac{AE_F}{A} \simeq \frac{3}{5} E_F \simeq 22MeV$$

Asymmetry term of the LDM model

Starting from equation $T(A) = T_p(Z) + T_n(N) = \frac{3}{5}[NE_F(n) + ZE_F(p)]$

Let's define $\epsilon = N - Z$ and use $E_F = \frac{\hbar^2}{2Mr_0^2}\left(\frac{9\pi}{8}\right)^{2/3}$

The kinetic energy is $T(A) = \frac{3}{10}E_F A\left[\left(1+\frac{\epsilon}{A}\right)^{5/3} + \left(1-\frac{\epsilon}{A}\right)^{5/3}\right]$

And can be expanded in powers of ϵ/A

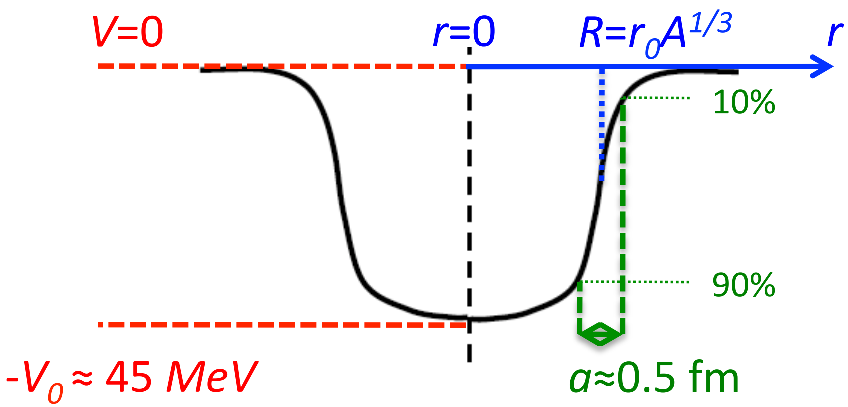
$$T(A) = \frac{3}{5}E_F A + \frac{1}{3}E_F\frac{(N-Z)^2}{A} + \ldots$$

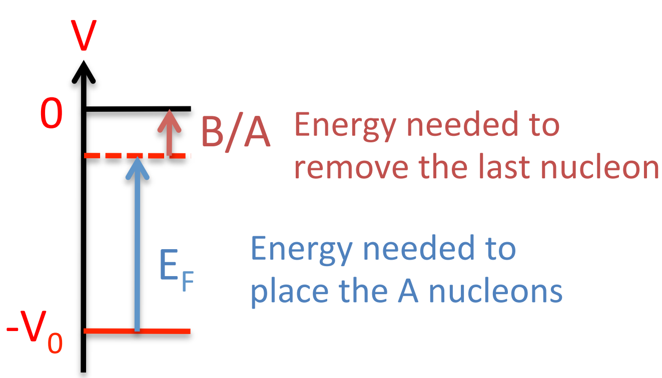
- ✓ The first term is a contribution to the volume term.
- ✓ The second term contributes to the asymmetry term; but it accounts only for 12.5 MeV out of the 23 MeV of u_T.
  A proton and a neutron with overlapping wavefunctions will have a greater strong interaction between them and stronger binding energy. This makes it energetically favorable for protons and neutrons to have the same quantum numbers, and thus increase the energy cost of asymmetry between them.

3- Nuclear mean potential

- Nucleons move in a net potential that represents the average effect of interactions with the other nucleons in the nucleus.
- Short range and saturation of nuclear interaction → interaction intensity proportional to the nuclear density
- Woods-Saxon potential

$$V_{WS}(r) = -\frac{V_0}{1 + e^{\frac{r-R}{0.228\,a}}}$$

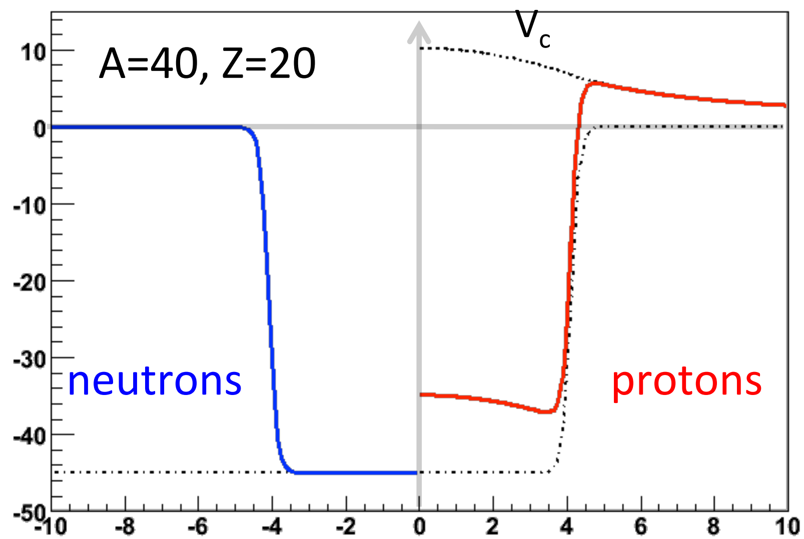




$$V_0 \approx E_F + B/A \approx 37 + 8 \approx 45 \text{ MeV}$$

- In fact, the protons also feel the coulomb potential:

$$V_c(r) = \begin{cases} (Z-1)\frac{\alpha\hbar c}{r} & \text{pour} \quad r \geq R \\ (Z-1)\frac{\alpha\hbar c}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right] & \text{pour} \quad r < R \end{cases}$$



- We will see later an additional interaction term: spin-orbit coupling

$$V_{LS} = +g\vec{L}.\vec{S} \quad \text{with} \quad g = -24\,A^{-2/3}$$ (There are other parameterizations)

$$\text{where} \begin{cases} \vec{L} & \text{is the orbital angular momentum of the nucleon} \\ \vec{S} & \text{is the spin of the nucleon} \end{cases}$$

- The mean nuclear potential is:

$$V_{\text{nucl}} = V_{\text{WS}}(r) + V_{\text{c}}(r) + g\,\vec{L}.\vec{S}$$

4- Shell Model

- Basic idea: solve Schrödinger equation $H_0\Psi = E\Psi$ with

$$H_0 = \sum_{\alpha=1}^{A}\left[T_\alpha + V(r_\alpha) + g\vec{L}_\alpha.\vec{S}_\alpha\right]$$

- Treat each nucleon independently and solve A one-body problems, i.e. Solve A decoupled Schrödinger equations for the nuclear potential:

$$\left[T_\alpha + V(r_\alpha) + g\vec{L}_\alpha.\vec{S}_\alpha\right]\psi_\alpha(\vec{r}_\alpha) = \epsilon_\alpha\psi_\alpha(\vec{r}_\alpha)$$

  - $\psi_\alpha(\vec{r}_\alpha)$: wavefunction of the nucleon α
  - ϵ_α: Energy of the nucleon α

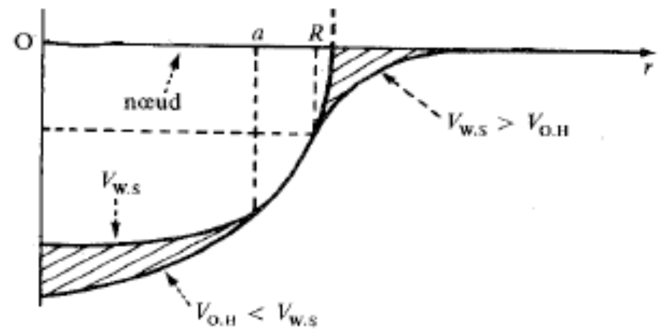
- For the nucleus:

$$E = \sum_{\alpha=1}^{A}\epsilon_\alpha$$

$$\Psi(\vec{r}_1, \vec{r}_1, ..., \vec{r}_A) = \left[\prod_{\substack{n=1 \\ \text{antisymetric}}}^{N}\psi_n(\vec{r}_n)\right]\left[\prod_{\substack{p=1 \\ \text{antisymetric}}}^{Z}\psi_p(\vec{r}_p)\right]$$ ⇒Slater determinant

Approximations

- Woods-Saxon potential : non analytic solutions
  - ✓ either numerical resolution
  - ✓ either perturbative treatment
- Possible simplification with the same symmetries (central potential)
  - Square well
  - Harmonic oscillator



$$V_{HO} = -V_0 \left[1 - \left(\frac{r}{R}\right)^2\right] = -V_0 + \frac{1}{2} m\omega_0^2 r^2$$

$$\omega_0 = \sqrt{\frac{2V_0}{mR^2}}$$

$$\hbar\omega_0 = \frac{\hbar c}{R}\sqrt{\frac{2V_0}{mc^2}} \simeq 46\, A^{-1/3}\ \mathrm{MeV}$$

  - Side effect correction: add a contribution proportional to r^4.
    It is possible to show that this is equivalent to add a term proportional to L^2

$$V_{nucl} = V_{HO} + D\,\vec{L}^2 \quad \text{with} \quad D < 0$$

4.1 Solutions for a central potential

Solve Schrödinger equation for a symmetric central potential:

- Separate equations for angular and radial coordinates
- Radial equation:

$$\left[-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{\hbar^2\ell(\ell+1)}{2mr^2} + (V_{nucl} - E)\right] U_{n\ell}(r) = 0$$

$$\text{with } U_{n\ell} = rR_{n\ell}$$

- Wave function: $\psi_\alpha(\vec{r}_\alpha, \vec{s}_\alpha) = R_{nl}(r_\alpha)\, Y_\ell^{m_\ell}(\theta_\alpha, \phi_\alpha)\, \chi_s^{m_s}(\vec{s}_\alpha)$
  - Radial quantum number: $n=0,1,2,\ldots,\infty$
  - Orbital angular momentum : $\ell=0,1,2,\ldots,\infty$ *(nb: any ℓ given n)*
  - Orbital magnetic quantum number: $m_\ell=-\ell,\ldots+\ell$
  - Nucleon spin : $s=1/2$
  - Nucleon spin quantum number: $m_s=-1/2,1/2$

- Energy: in the most general case depends of *n* and ℓ.
- Level degeneracy: $g_{n\ell} = (2s+1)(2\ell+1) = 2(2\ell+1)$

Special case : 3D Harmonic oscillator plus side effect correction

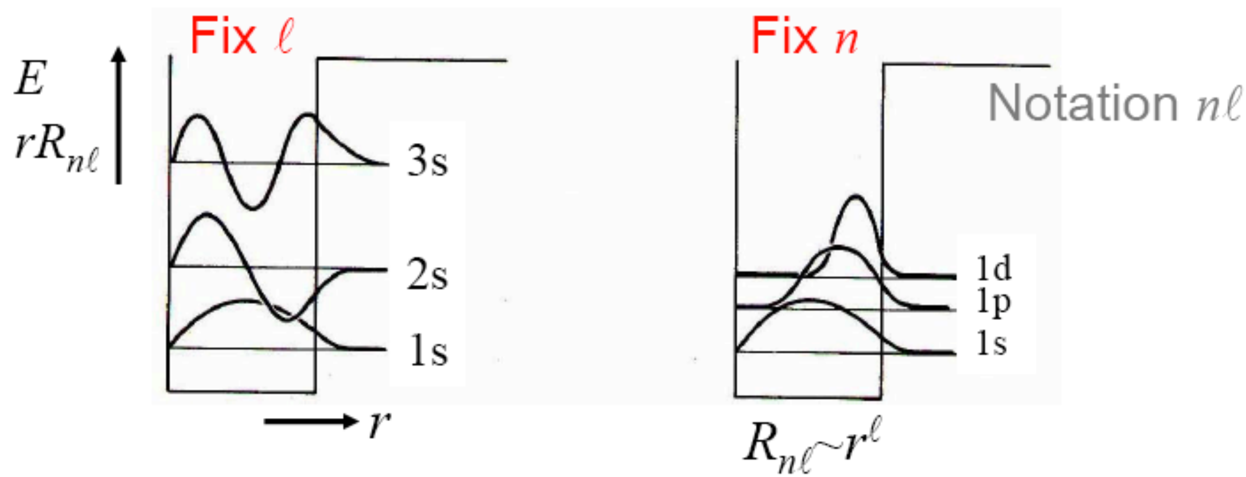
- Cartesian radial wavefunctions: Hermite polynomials
  - Wavefunction: $\psi_{n_x,n_y,n_z}(x,y,z) = C\ H_{n_x}(x)H_{n_y}(y)H_{n_z}(z)$
  - Energy: $E_N = (n_x + n_y + n_z + 3/2)\,\hbar\omega_0 = (N + 3/2)\,\hbar\omega_0$
  - Degeneracy: $g_N = (2s+1)\,(N+1)(N+2)/2$
- Spherical radial wavefunctions: Laguerre polynomials (good symmetry)
  - Wavefunction: (cf TD)
  - Energy: $E_{n\ell} = (2n + \ell + 3/2)\hbar\omega_0 = (N + 3/2)\hbar\omega_0$
    with $N = 2n + \ell$: quantum principal number — accidental ℓ degeneracy
  - Degeneracy: $g_N = (2s+1)\,(N+1)(N+2)/2$
- With side effect correction $V = V_{HO} + D\vec{L}^2$
  - Energy: $E_{n\ell} = (N + 3/2)\hbar\omega_0 + D\ell(\ell+1)\hbar^2$
  - Degeneracy: $g_{n\ell} = (2s+1)(2\ell+1) = 2(2\ell+1)$

  breaks ℓ degeneracy

- Energy levels increase with *n* and *ℓ*
- Spectroscopic notation (*n*+1),*ℓ*

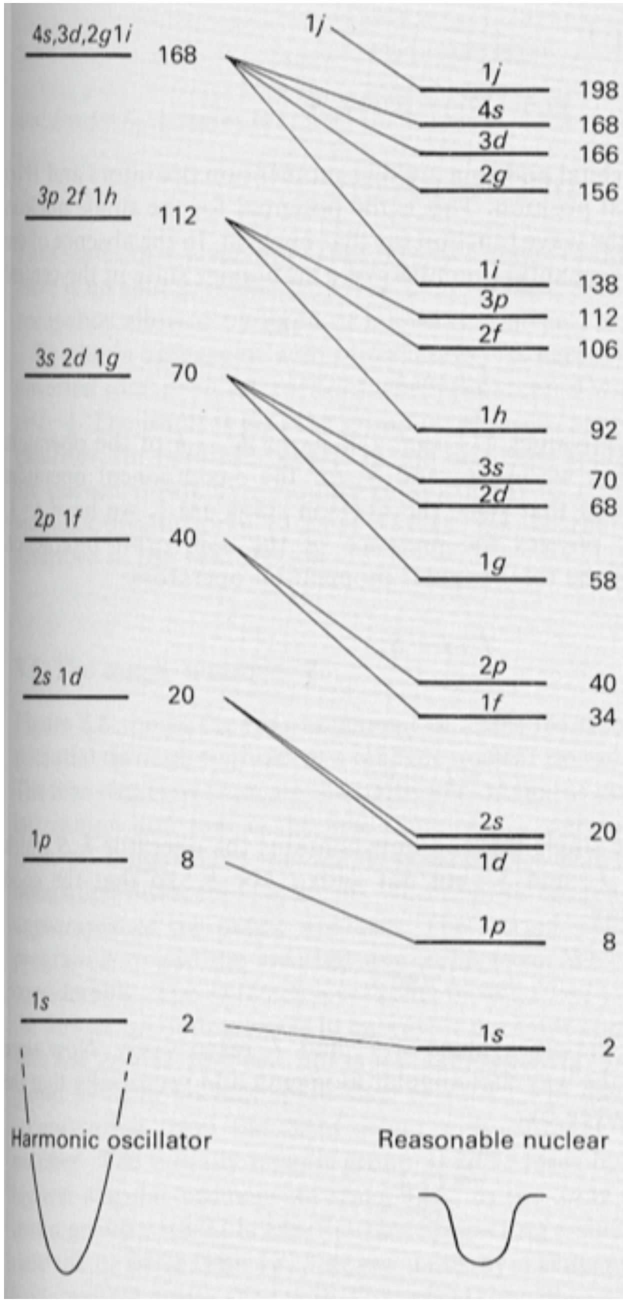
ℓ	0	1	2	3	4	5	6
notation	s	p	d	f	g	h	i

Example



As n increases,
$rR_{n\ell}$ has more nodes, greater curvature and E increases.

As ℓ increases,
$rR_{n\ell}$ has greater curvature and E increases.



- Level degeneracy : $2(2\ell+1)$

- Unable to reproduce the magic numbers:
  2, 8, 20, 40,58,…

3D Harmonic oscillator states

N	$E_N/\hbar\omega$	g_N	Σg_N	n+1,ℓ	parity
0	3/2	2	2	1s	+
1	5/2	6	8	1p	-
2	7/2	12	20	1d,2s	+
3	9/2	20	40	1f,2p	-
4	11/2	30	70	1g,2d,3s	+
5	13/2	42	112	1h,2f,3p	-
6	15/2	56	168	1i,2g,3d,4s	+

4.2 Spin-orbit interaction

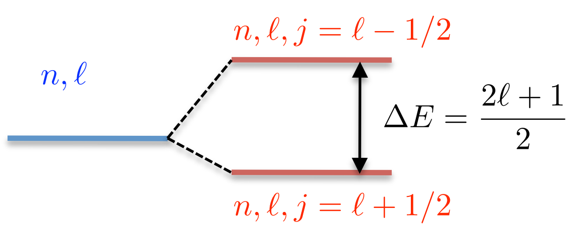
- Mayer and Jensen (1949) included spin-orbit potential to explain magic numbers: $V(r) = V_{\mathrm{central}}(r) + g\,\vec{L}.\vec{S}$
  where the spin-orbit coupling *g* is negative.
  Origin completely different from atomic spin-orbit:
  - Atomic physics : relativistic effect
  - Nuclear physics : 1st order effect in the mean potential. The nucleon-nucleon interaction contains a term depending on the relative spin state of the nucleons but also on the speed of the nucleons (L=r×mv)
- The spin-orbit interaction splits ℓ levels into j values

$$\vec{J} = \vec{L} + \vec{S} \qquad \vec{L}.\vec{S} = \frac{1}{2}[\vec{J}^2 - \vec{L}^2 - \vec{S}^2] \qquad \langle\psi|\vec{L}.\vec{S}|\psi\rangle = \frac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]$$

(decoupled base→coupled base)

- For a single nucleon *s=½* and *j=ℓ±½*

$$\begin{cases} j = \ell - 1/2 & \langle\vec{L}.\vec{S}\rangle = -\frac{\ell+1}{2} & E = E_{\mathrm{n}\ell} - g\frac{\ell+1}{2} \\ j = \ell + 1/2 & \langle\vec{L}.\vec{S}\rangle = \frac{\ell}{2} & E = E_{\mathrm{n}\ell} + g\frac{\ell}{2} \end{cases}$$



since g<0, layer ℓ+½ is lowered and layer ℓ-½ is raised

- **Wavefunction:**

$$\psi_{n\ell sjm_j}(\vec{r},\vec{s}) = R_{n\ell}(r)\mathcal{Y}^{\ell s}_{jm_j}(\Omega,\vec{s})$$

- **Energy** (HO case):

$$En\ell j = \hbar\omega_0(2n + l + 3/2) + D\ell(\ell+1) + \begin{cases} -g\frac{\ell+1}{2} & \text{for } j = \ell - 1/2 \\ +g\frac{\ell}{2} & \text{for } j = \ell + 1/2 \end{cases}$$

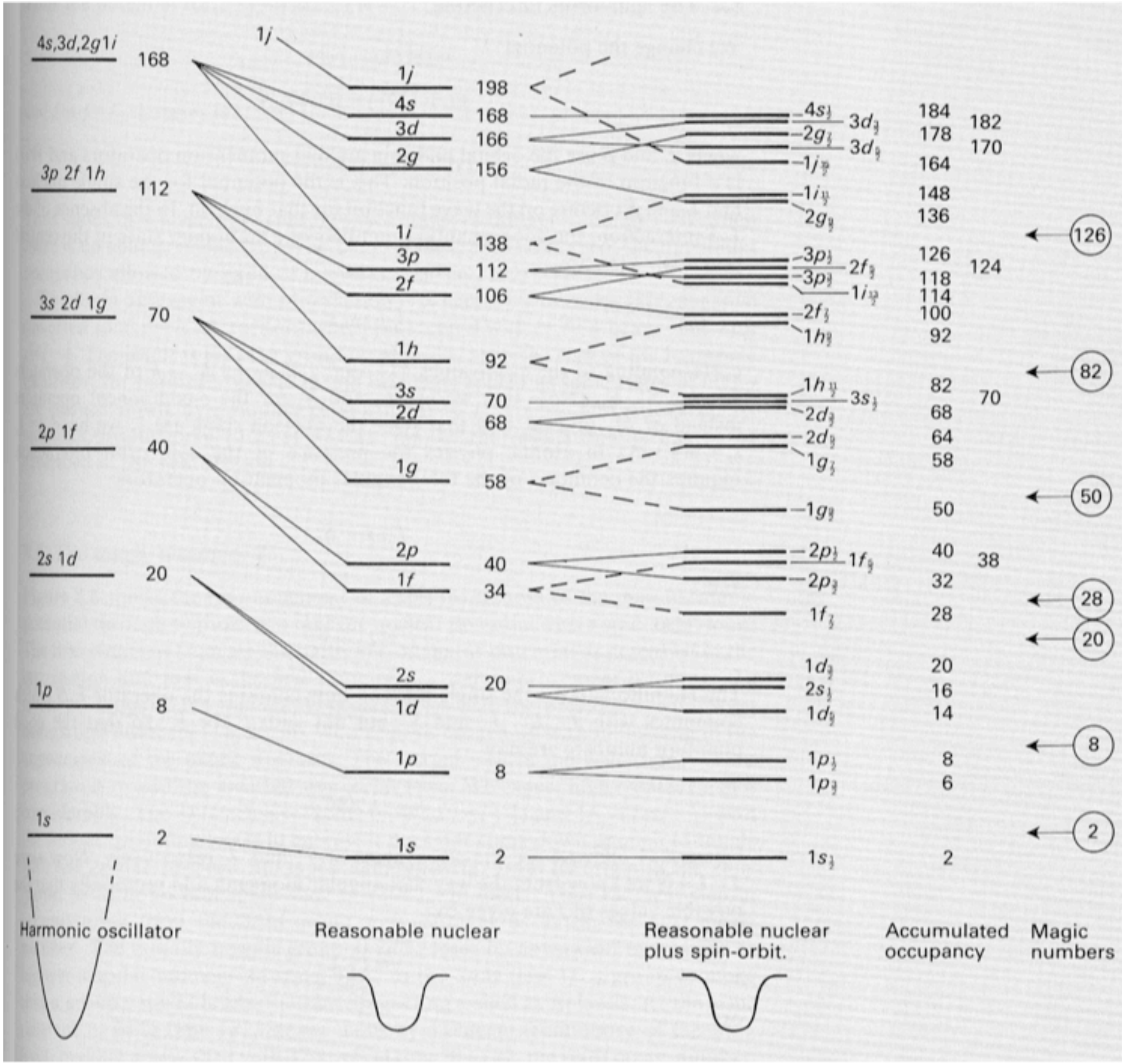
- **Degeneracy:**

$$g_j = 2j + 1$$

- **Spectroscopic notation:** $(n+1)\ \ell_j$

ℓ	0	1	2	3	4	5	6
notation	s	p	d	f	g	h	i

- For a proton level, the notation is preceded by π, for a neutron by ν examples: $\pi 2p_{3/2}$ or $\nu 1g_{9/2}$

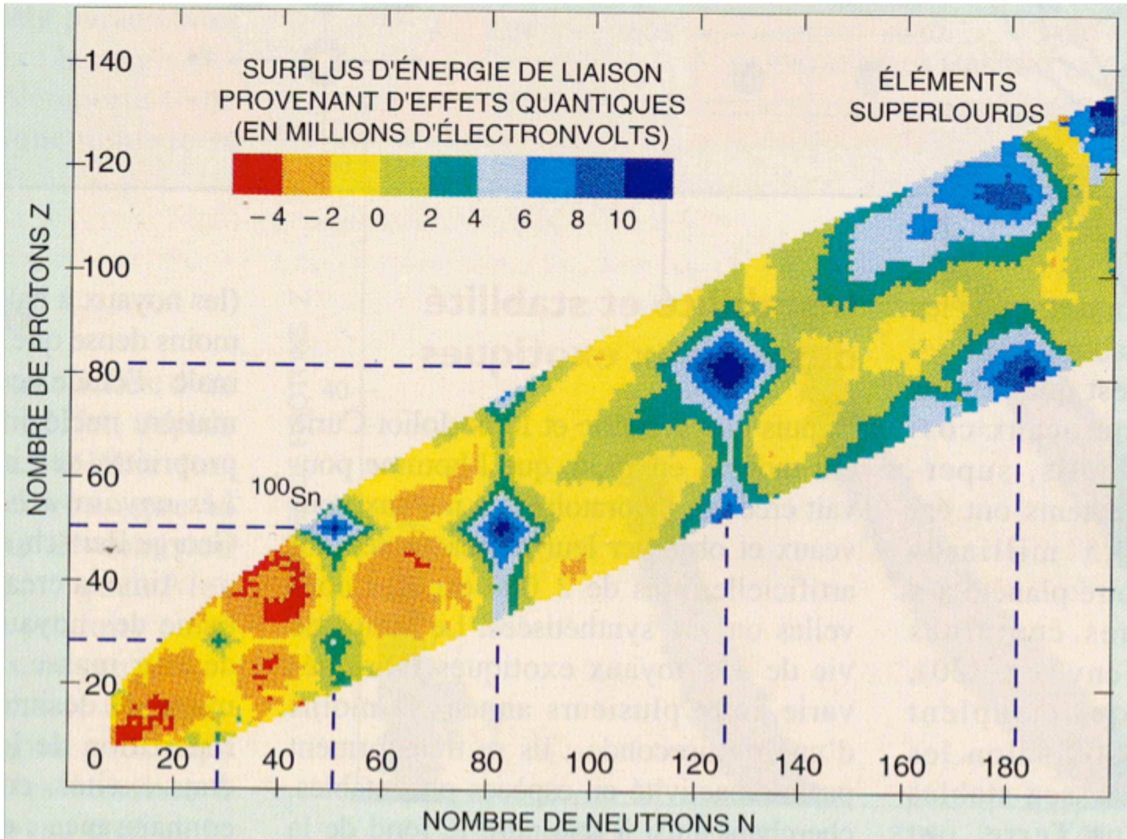


Explain magic numbers:
2,8,20,28,50,82,126

Experimentally, after 82 protons and neutrons magic numbers differ:
- p: 114 and 126
- n: 126 and 184

strong coulomb effect

Super heavy nuclei ?



Pour certains nombres de protons ou de neutrons, les nombres magiques, la stabilité du noyau est accrue (couleurs bleues). Ces effets de couche se superposent à la répulsion électrostatique et à l'attraction forte.

Some very heavy doubly magic nuclei are predicted:

$$^{289}_{114}\mathrm{X} \text{ and } ^{310}_{126}\mathrm{X}$$

4.3- Predictions of the shell model

✓ 4.3.1- Magic numbers *(constructed in this goal)*

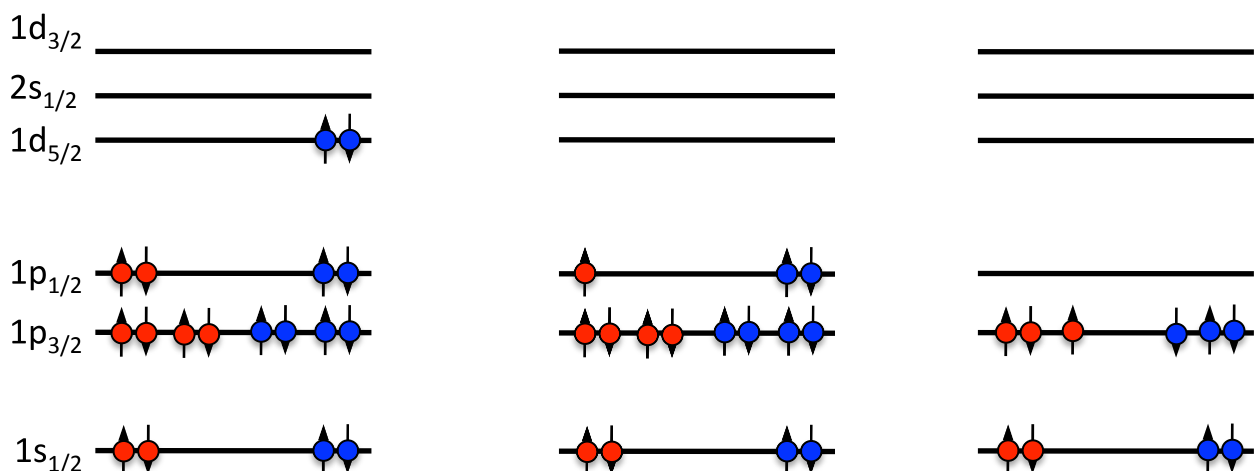
✓ 4.3.2- Spin (J) and parity (π) of the ground state

(near closed shells → Mayer-Jensen rules)

- Even-even nuclei: $J^\pi = 0^+$
- Odd-A nuclei: J^π given by the unpaired nucleon or hole; $\pi = (-)^\ell$
- Odd-odd nuclei: compute J from unpaired proton and neutron, the apply j-j coupling

$$|j_p - j_n| \leq J \leq j_p + j_n \qquad \pi = (-)^{l_p} \times (-)^{l_n}$$

Examples



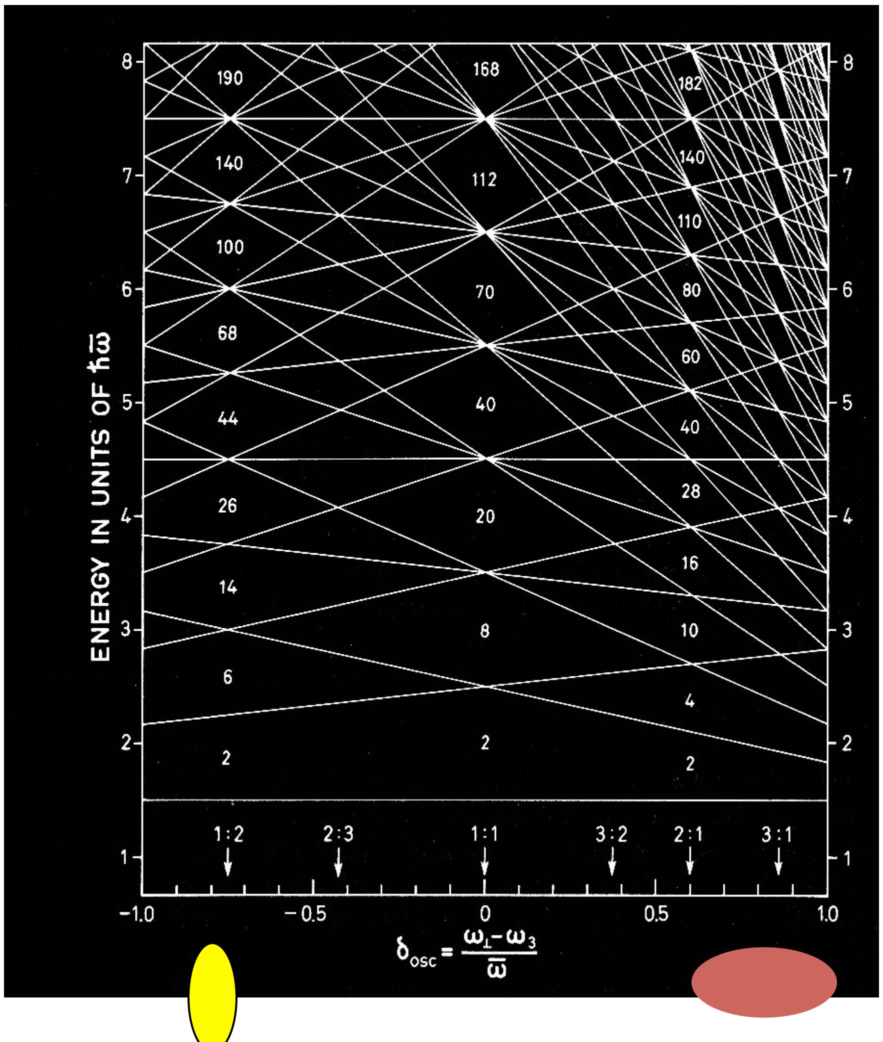
$^{18}_{8}\mathrm{O}$ — $J^\pi = 0^+$, exp: 0^+

$^{15}_{7}\mathrm{N}$ — $J^\pi = \frac{1}{2}^-$, exp: $\frac{1}{2}^-$

$^{10}_{5}\mathrm{B}$ — $j_p = 3/2$ $j_n = 3/2$; $J^\pi = 0^+, 1^+, 2^+, 3^+$; exp: 3^+

✓ 4.3.3 Spin and parity away from closed shells

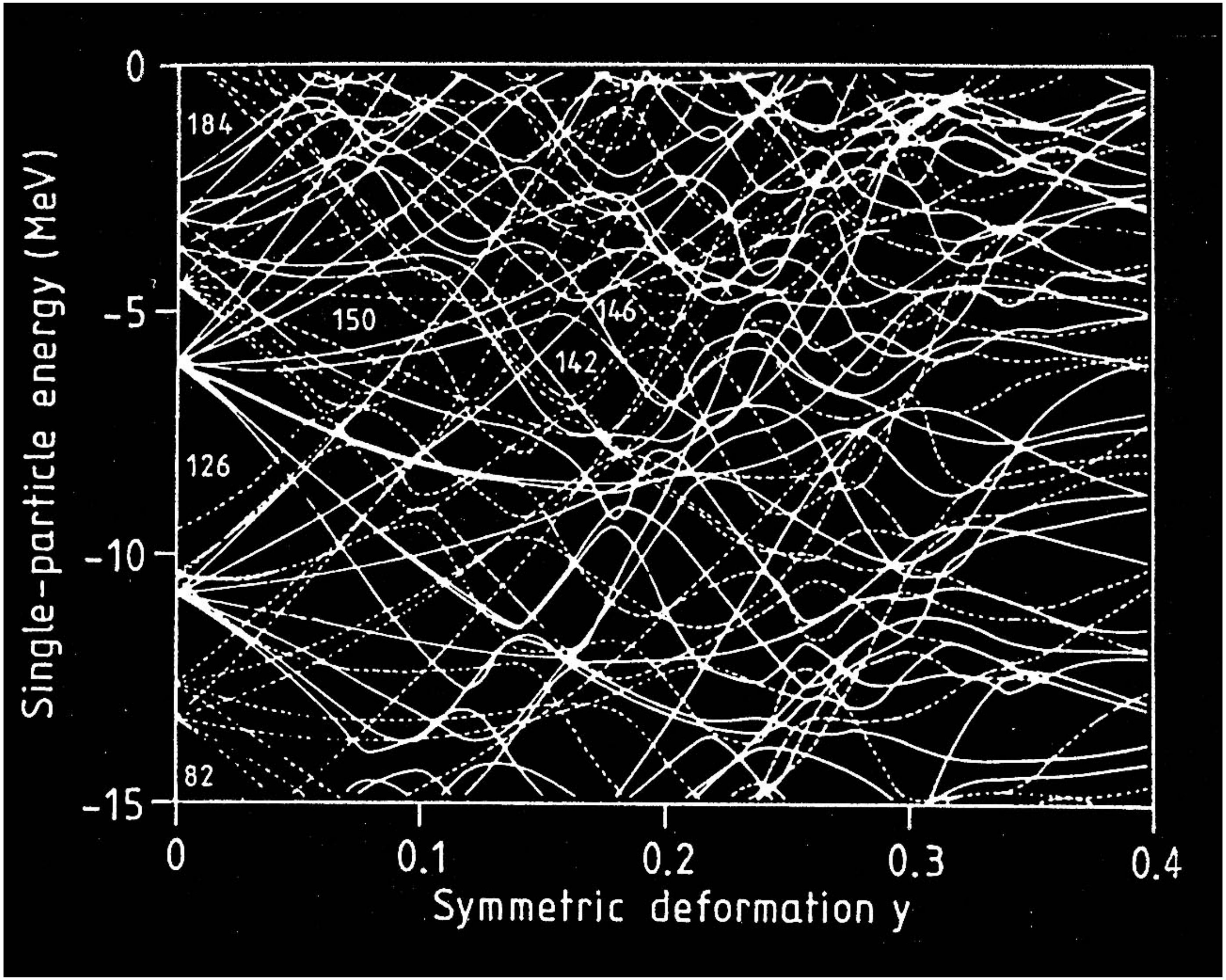
more than one nucleon can contribute and electric quadrupole Q can be large → V(r) no longer symetric



Deformed shell model: Example with a deformed harmonic oscillator

j_z degeneracy partially removed

La réalité de l'évolution des niveaux …



… complexité !

✓ 4.4.4- Magnetic moments

- Nuclear magnetic dipole moments arise from the intrinsic spin magnetic dipole moments of the protons and neutrons in the nucleus and from currents circulating in the nucleus due to the motion of the protons:

$$\vec{\mu} = \frac{\mu_N}{\hbar} \sum_i [\, g_\ell \vec{\ell}_i + g_s \vec{s}_i \,]$$ where $\mu_N = \frac{e\hbar}{2m_p}$ is the nuclear magneton

and the proton and neutron g-factor

$$p: \quad g_\ell = 1 \quad g_s = 5.586$$
$$n: \quad g_\ell = 0 \quad g_s = -3.82$$

- Using the projection theorem in QM, it is possible to show that μ is proportional to J_z, and thus has m_J possible values. The greatest of these values is called the Nuclear magnetic dipole moment μ:

$$\mu = g_J \, \mu_N \, J$$

- The Nuclear magnetic dipole moment has m_J values
  - J is the the total angular momentum of the nucleon, also called "spin" of the nucleus
  - g_J is the nuclear g-factor

$$\mu = g_J \, \mu_N \, J$$

- All even-even nuclei have μ=0 since J=0

- For odd-A nuclei, μ is the one of the unpaired nucleon.
  it is possible to show (see TD):

$$g_J = \frac{1}{2j(j+1)} \{ g_\ell [\ell(\ell+1) + j(j+1) - s(s+1)] + g_s [s(s+1) + j(j+1) - \ell(\ell+1)] \}$$

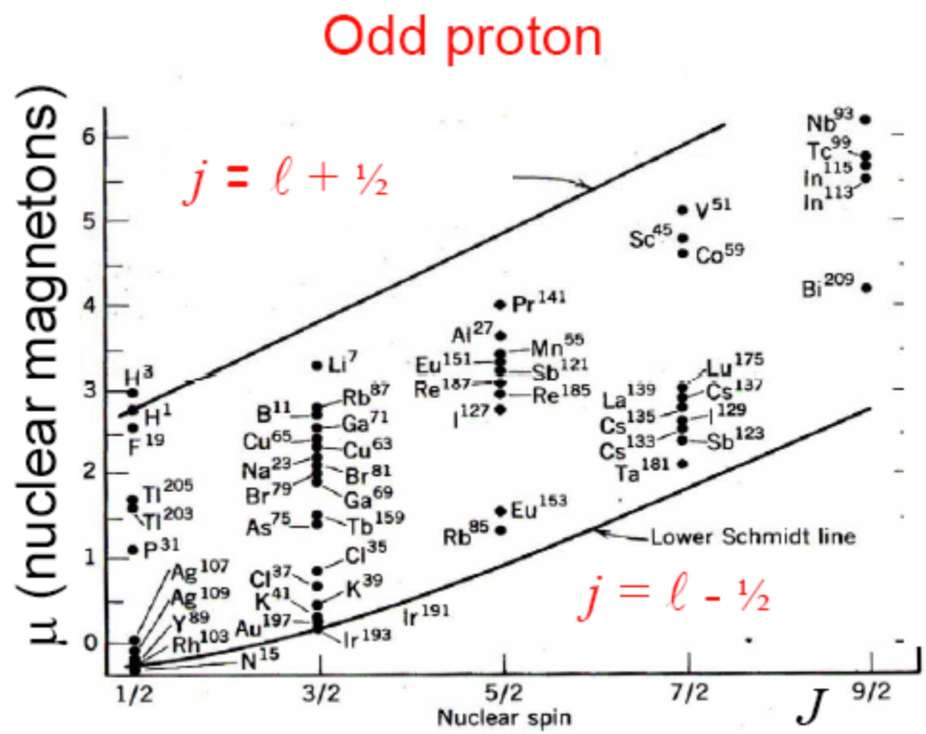
$$\text{since } s = 1/2, \; j = \ell \pm 1/2 \Longrightarrow g_J = g_\ell \pm \frac{g_s - g_\ell}{2\ell+1}$$

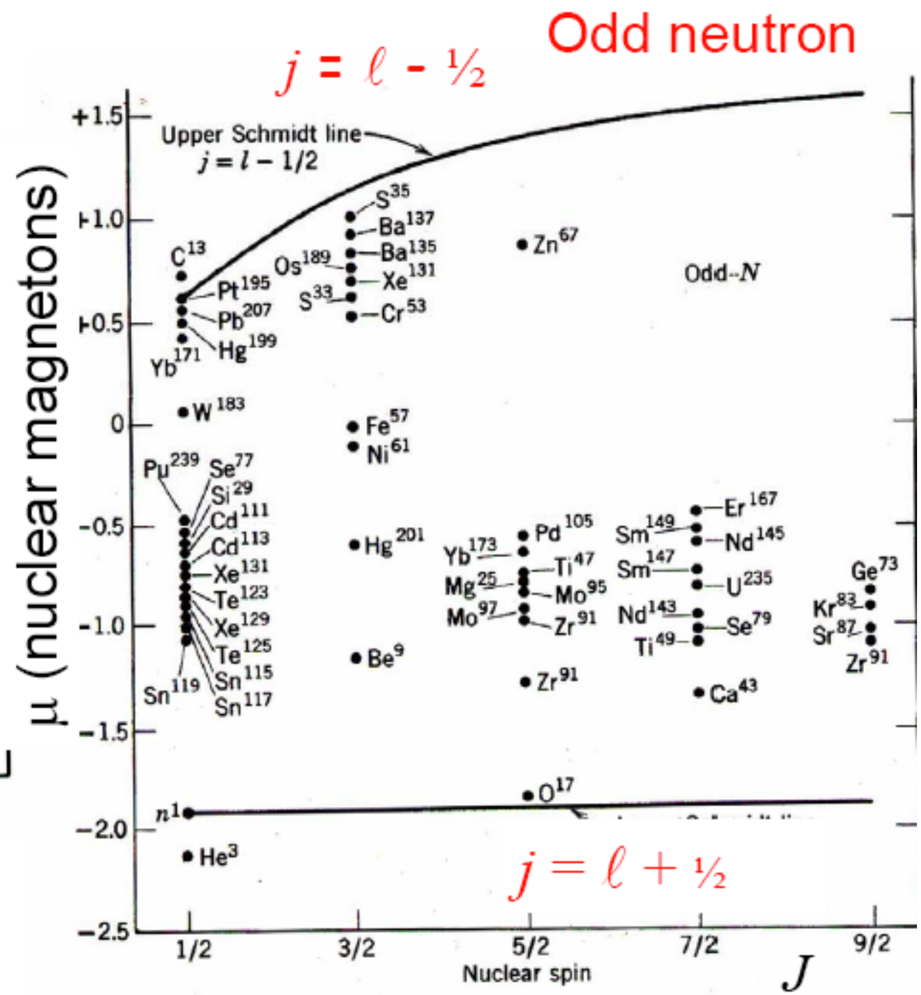
where g_l and g_s are chosen according to the type of the single nuclei (n or p)

The 4 different combinations (p,n) and (j=l±½) are called the Schmidt Limits

- For odd-odd nuclei, the magnetic moment is due to both unpaired nucleons, one has to use a j-j coulping.

Schmidt Limits compared to experimental data





5- Excited states of nuclei

There are three kinds of excitation in the nuclear spectra:

- ✓ Single nucleon excited states
- ✓ Vibrational excited states
- ✓ Rotational excited states

All these excitations decay by γ emission.

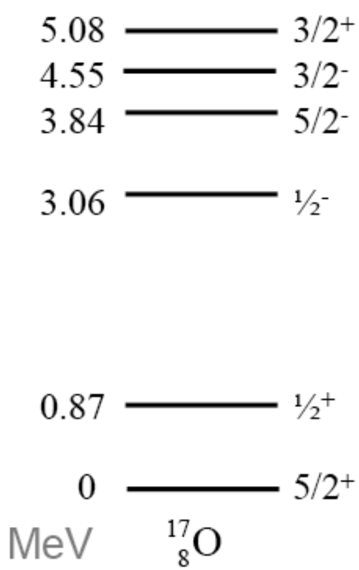
5.1- Single nucleon excited states

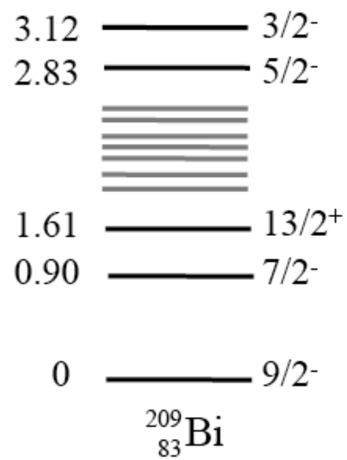
Can be predicted from the Shell Model.

Successful for small excitations of odd-A nuclei near closed shells.

- Particle excitation
- Hole excitation

Examples





$1s_{1/2}$ /2
$1p_{3/2}$ $1p_{1/2}$ /8
$1d_{5/2}$ $2s_{1/2}$ $1d_{3/2}$ /20
$1f_{7/2}$ /28
$2p_{3/2}$ $1f_{5/2}$ $2p_{1/2}$ $1g_{9/2}$ /50
$1g_{7/2}$ $2d_{5/2}$ $2d_{3/2}$ $3s_{1/2}$ $1h_{11/2}$ /82
$1h_{9/2}$ $2f_{7/2}$ $1i_{13/2}$ $3p_{3/2}$ $2f_{5/2}$ $3p_{1/2}$ /126
$2g_{9/2}$ $1i_{11/2}$ $1j_{13/2}$ …

5.2- Collective excitations

Vibrational and rotational motion involve the collective motion of the nucleons in the nucleus. These collective motions can be incorporated in the shell model by replacing the symmetrical potential with a time dependant potential describing the shape deformations.

In these case, we will only consider even-even nuclei

- Ground state: $J^{\pi}=0^{+}$, lowest excited state (almost always): $J^{\pi}=2^{+}$
- Classification:

A	E(2⁺)	Type
30 - 150	~ 1 MeV	Vibrational
150 – 190 (rare-earth)	~ 0.1 MeV	Rotational
>220 (actinides)	~0.1 MeV	Rotational

Deformed regions mid-distance between magic numbers

Non examinable

5.2.1- Nuclear vibrations

- Vibrational excited states occur when a nucleus oscillate around a spherical equilibrium shape. The form of the excitations can be represented by a multipole expansion (like the underlying nuclear shapes).

Monopole — Incorporated into the average radius

Dipole — Involves a net displacement of c.m. ⇒ cannot result from action of nuclear forces.

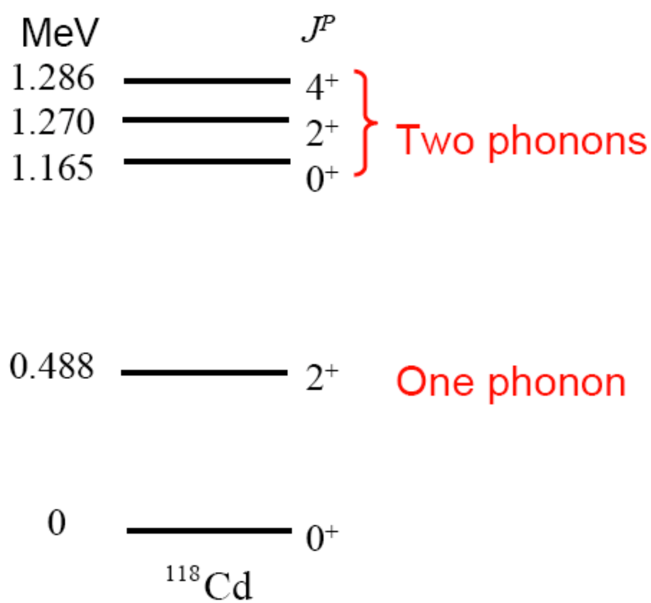
Quadrupole

Octupole

- Quadrupole oscillations are the lowest order nuclear vibrational mode. The quanta of vibrational energy are called Phonons

Non examinable

- A quadrupole phonon carries 2 units of angular momentum and has even parity.
  - First excited state: $J^\pi=2^+$
  - Second excited state: $J^\pi=0^+,2^+,4^+$ (in practice not always degenerate)
  - Ratio E(2nd excited)/E(1st excited)≈2
- An octupole phonon carries 3 units of angular momentum and has an odd parity. First excited octupoles states often lies near the quadrupole 2 phonon triples states.
- Example:

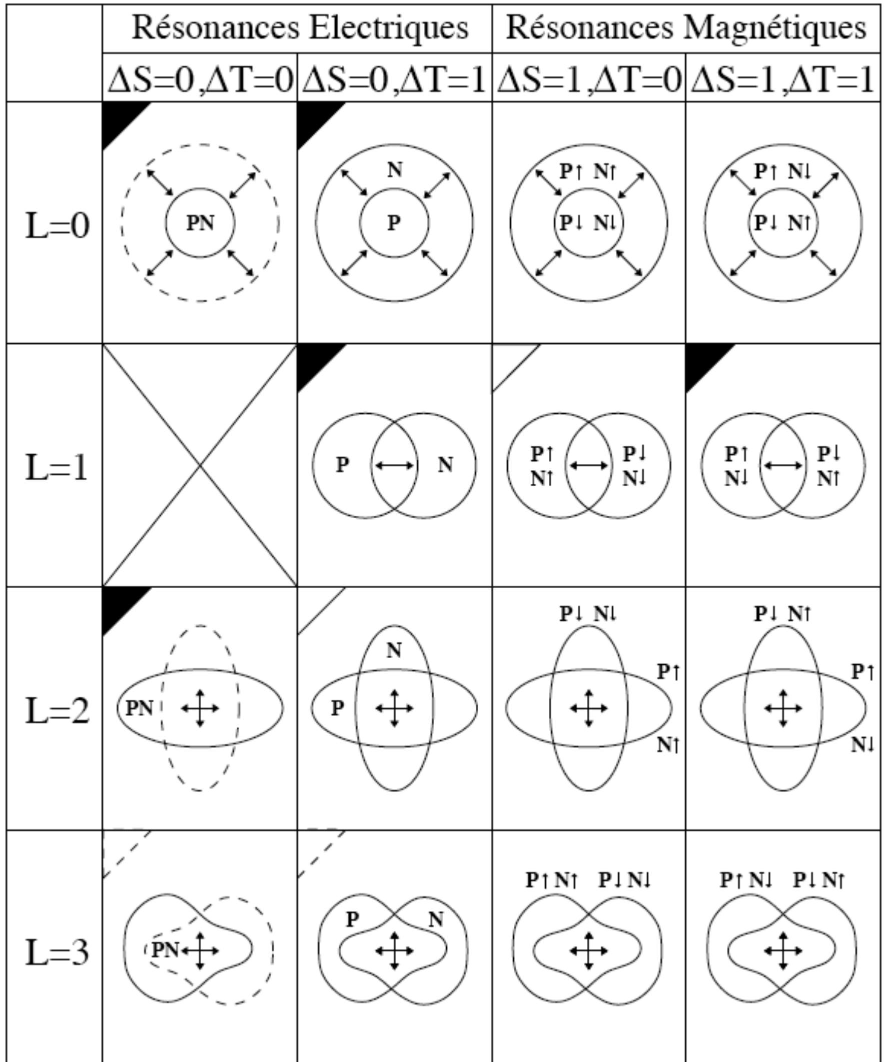


"vibrational band"

E(2 phonons)/E(1 phonon) ≈ 2

Non examinable

- Giant resonance: collective oscillation of all protons against all neutrons in a nucleus. Continuous γ-ray spectrum, E>10 MeV



Non examinable

5.2.2- Nuclear rotations

- Collective rotational motion can only be observed in deformed nuclei: (rotation around a symmetry axis is not observable)
- Rotational energy spectrum follows a law in J(J+1) where J is the spin of the nuclear state.

$$E_J = \frac{\hbar^2}{2\mathcal{I}} J(J+1)$$

$\mathcal{I}$ is the effective moment of inertia of the nucleus.

- The nucleus mirror symmetry restricts the sequence of rotational states to even values of angular momentum (not proved here):

  $J^\pi = 0^+, 2^+, 4^+, 6^+, \ldots$

  prediction → E(4+)/E(2+)=4(4+1)/2(2+1)=3.33

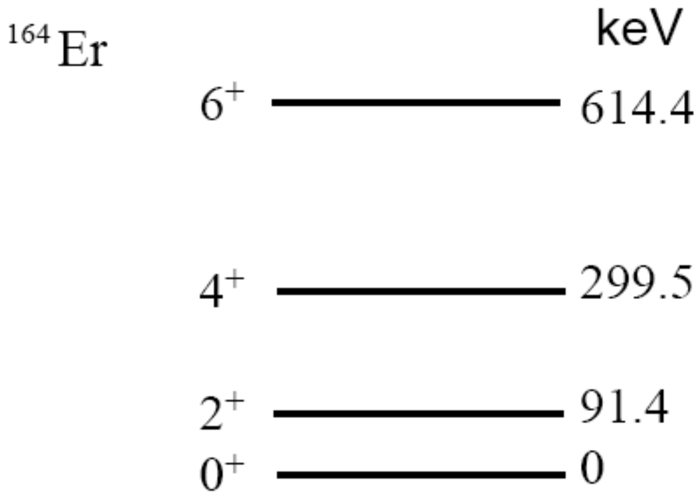
- $\mathcal{I}$ can be calculated from experimental data; find values lower than the rigid body value → only part of the nucleons are in collective motion.

  The rotational behavior is intermediate between the nucleus tightly bound and weakly bound → the strong force has not a long range

- $\mathcal{I}$ can change as J increase: centrifugal forces can cause a change in the deformation

Non examinable

Rotational spectrum of ^{164}Er

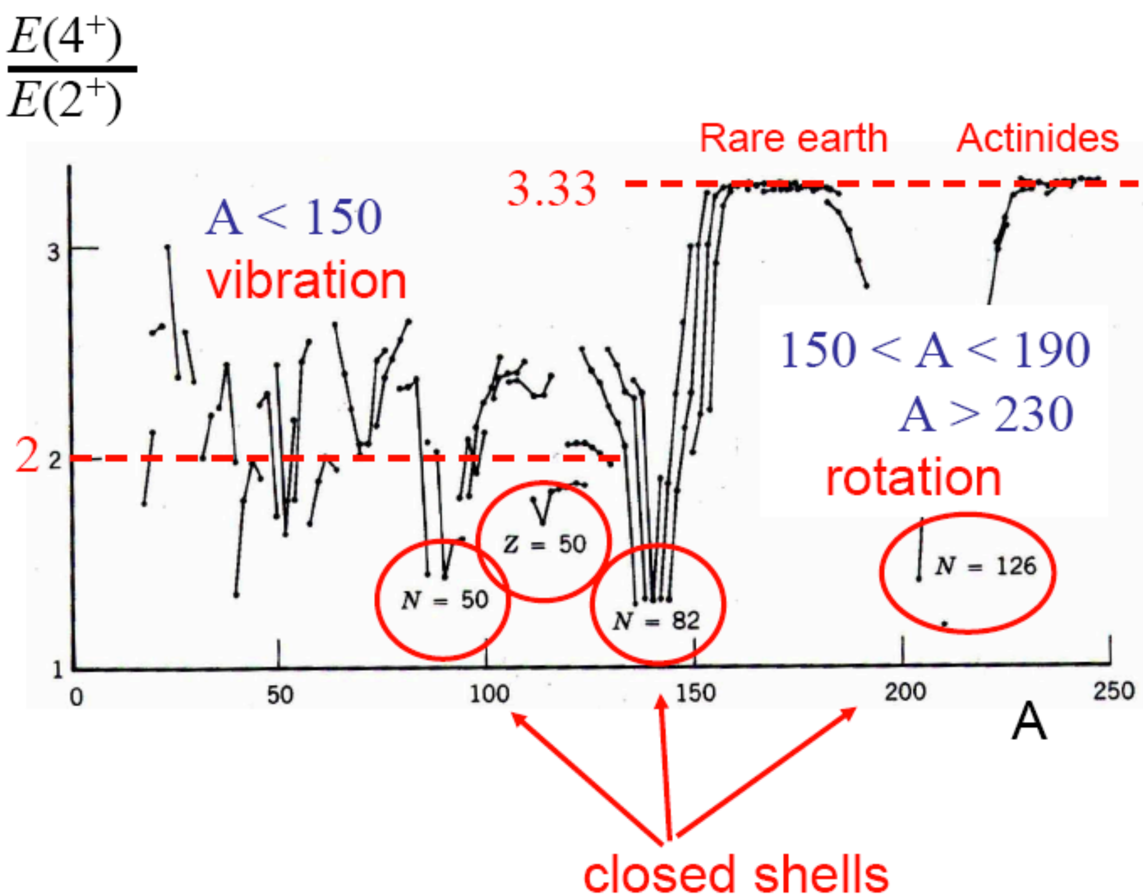


Observe $\frac{E(4^+)}{E(2^+)} = \frac{299.5}{91.4} = 3.28$

Predict $\frac{4(4+1)}{2(2+1)} = 3.33$

Non examinable

Ratio of 4+/2+ excitation energies for even-even nuclei



Exemple du ^{152}Dy

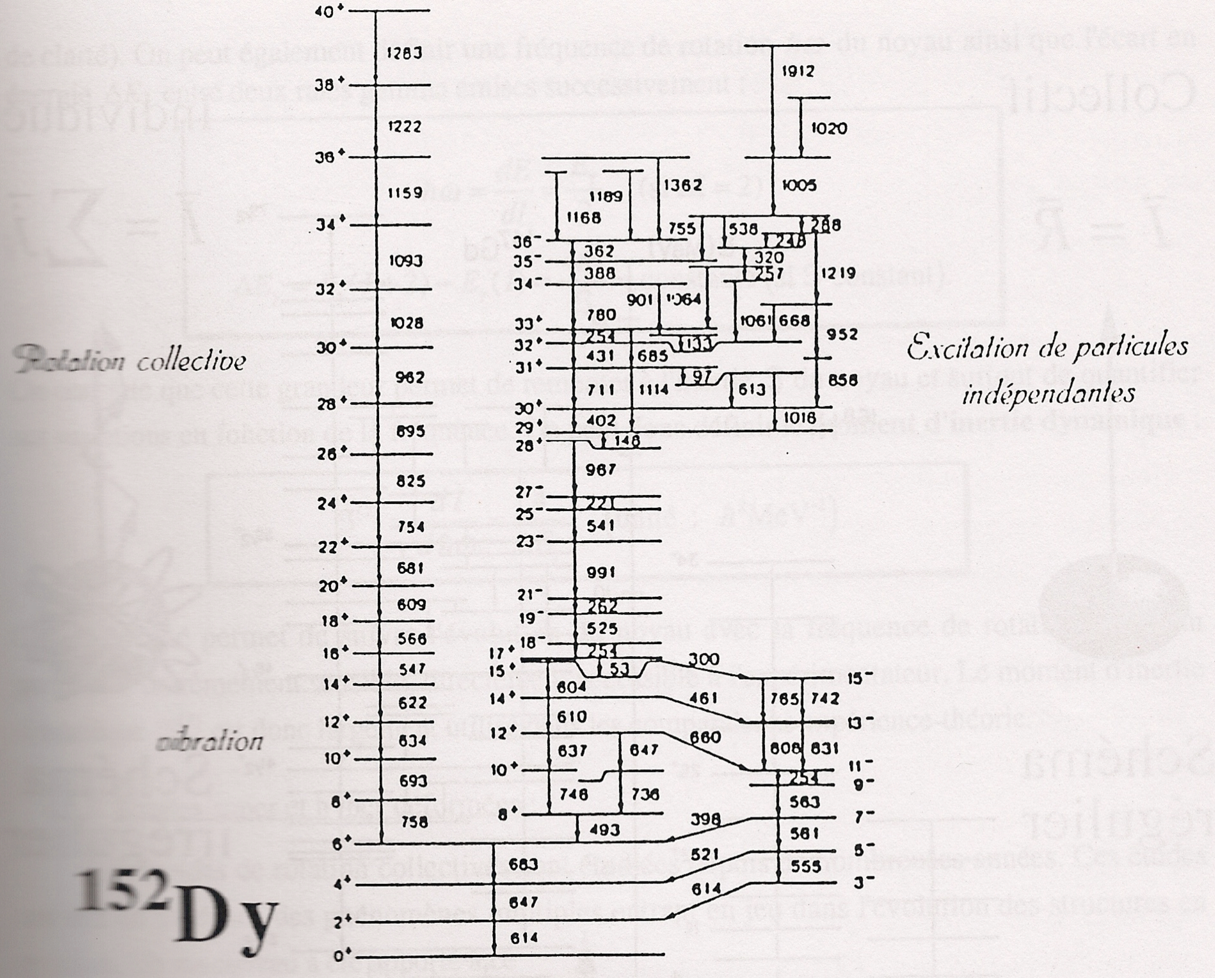


Figure I.6 : Schéma de niveaux excités de ^{152}Dy. On peut y observer, sur le côté droit des espacements de niveaux irréguliers liés à des excitation individuelles et sur la gauche un comportement régulier lié à des excitations collectives [Cu88].