Chapter 8

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Weak interactions

Outline/Plan

1. Introduction
2. Fermi's weak interaction theory
    1. Introduction
    2. Parity violation
    3. V-A theory
3. Applications to some processes
    1. Leptonic processes
    2. The quarks and the flavor mixing
    3. Neutrino properties

1. Introduction
2. Théorie de l'I.f. de Fermi
    1. Introduction
    2. La violation de la parité
    3. Théorie V-A
3. Applications à qqs processus
    1. Processus leptoniques
    2. Le secteur des quarks et le mélange de saveurs
    3. Les propriétés des neutrinos

1- Introduction

- Experimental observations :

  - *large lifetimes for some particles :*

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

  - *at 1st order the decay lengths (lifetimes^{-1}) are*

$$\Gamma \propto \left| T_{fi} \right|^2$$

=> low coupling constants governing the involved processes

1- Introduction

- Weak interactions affect all particles but their effects can be masked by the manifestation of strong or electromagnetic interactions.

- On the other hand the pions, which are the lightest hadrons, can not decay through strong interaction processes, only weak or e.m. processes:

$$\pi^0 \rightarrow \gamma\gamma \quad \pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$

- Other specific feature : neutrinos interact through weak interactions only.

1- Introduction

- Experimental measurements :

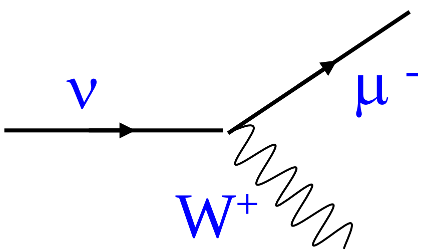
  - branching ratios :

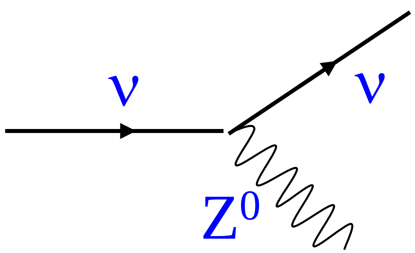
$$B\left(\frac{\mu \to e + \gamma}{\mu \to \text{all}}\right) < 10^{-10}$$

$$B\left(\frac{\mu \to e + e + e}{\mu \to \text{all}}\right) < 10^{-12}$$

  - reason : separate conservation of the leptonic numbers

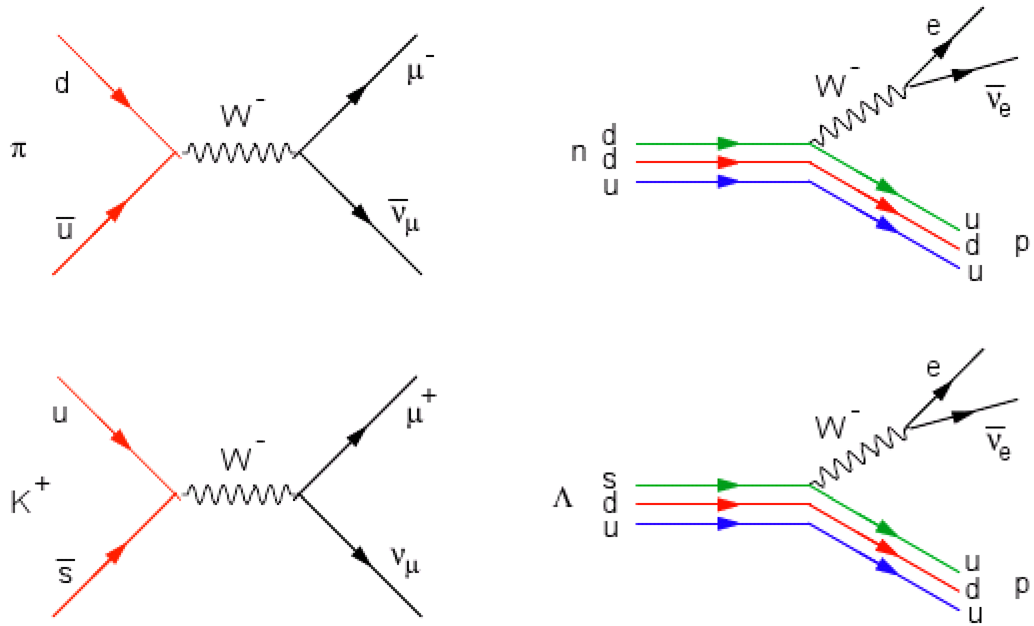
- Theoretical predictions (gauge theories) of charged and neutral currents :





1- Introduction

- Weak interactions classification :
  - *leptonic :* the gauge boson couple to leptons at both vertices
  - *semi-leptonic :* the gauge boson couple to leptons at one vertex and to quarks at the opposite vertex



$\Delta S=0$

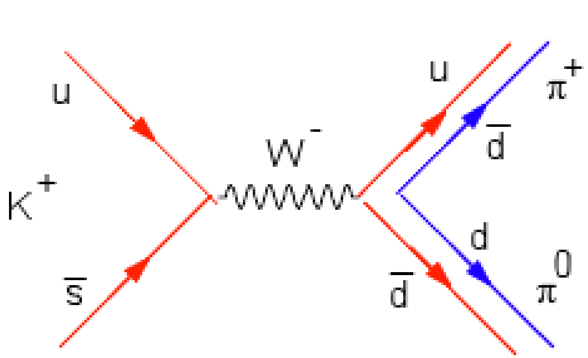
$\Delta S=1$

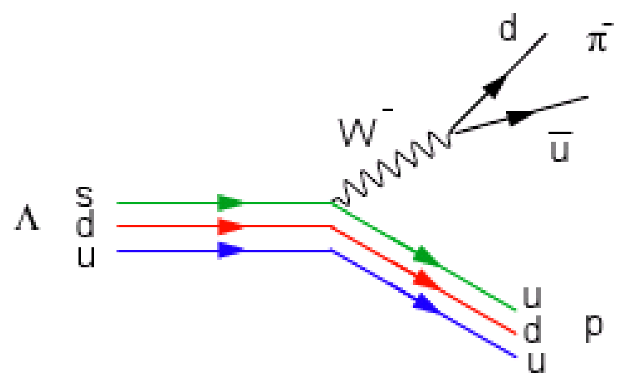
Warning: "strange" content may be changed…

1- Introduction

- Weak interactions classification :

  - *non-leptonic (hadronic) :* the gauge boson couple to quarks at both ends.





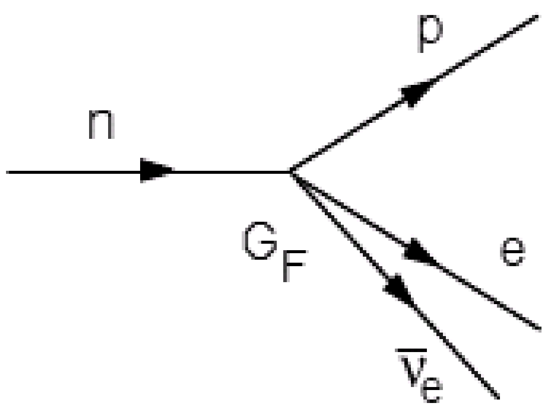
- In any case the W couples to fermions doublets *(f,f')* :

$$\begin{pmatrix} f \\ f' \end{pmatrix} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} d \\ u \end{pmatrix}, \begin{pmatrix} u \\ s \end{pmatrix} \cdots$$

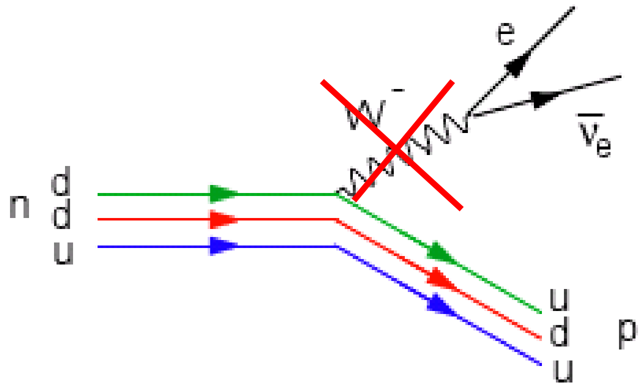
2-1- Introduction to Fermi's theory

- In 193O Fermi postulated his theory based on the assumption of a point-like 4-bodies interaction governed by a coupling constant called G_F.

- In that approximation the standard beta decay process is described by the following graph :



i.e.



2-1- Introduction to Fermi's theory

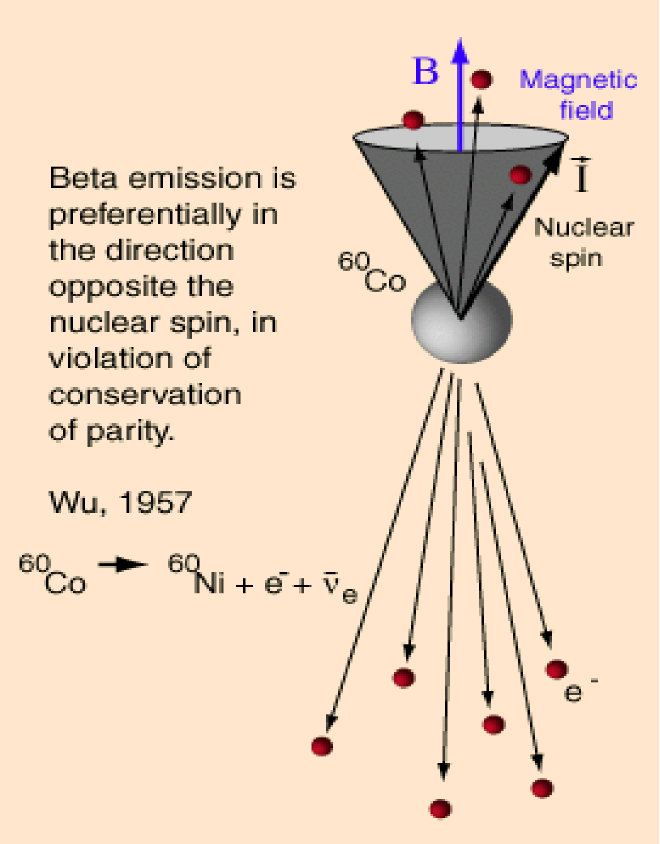
- By analogy with QED the transition matrix element may be written in the form of a current-current product :

$$T \propto G \underbrace{\left(\bar{u}_{\nu_e}\gamma^{\alpha}u_e\right)}_{\text{leptonic current}} \underbrace{\left(\bar{u}_n\gamma_{\alpha}u_p\right)}_{\text{hadronic current}}$$

- The main difference vs QED is

  the behavior wrt the parity symmetry, since e.m. interactions are invariant under the parity transforms while weak are not…

  The first experimental observations were performed in 1957 with ^{60}Co decays (Wu et al).



2-2- Parity violation

- Parity operation :

$$\vec{x} \xrightarrow{P} -\vec{x} \text{ i.e. } p^{\mu}\left(p^{0}, \vec{p}\right) \xrightarrow{P} p'^{\mu}\left(p^{0}, -\vec{p}\right)$$

- How does a Dirac spinor transform?

$$u \xrightarrow{P} u'$$

$$\left(\not{p} - m\right)u = 0 \xrightarrow{P} \left(\not{p}' - m\right)u' = 0$$

- Details : $\left(\not{p} - m\right)u = \left(\gamma^{0}p^{0} - \vec{\gamma}\vec{p} - m\right)u = 0 \quad \left(\to \gamma^{0}.\right)$

$$\Rightarrow \left(p^{0} - \gamma^{0}\vec{\gamma}\vec{p} - \gamma^{0}m\right)u = 0$$

$$\Rightarrow \left(p^{0} + \vec{\gamma}\vec{p} - m\right)\gamma^{0}u = 0$$

$$\Rightarrow \left(\not{p}' - m\right)\gamma^{0}u = 0 \quad \Rightarrow \boxed{u' = \gamma^{0}u}$$

2-2- Parity violation

- Summary :

$\bar{u}u$	S	$\varepsilon = +1$
$\bar{u}\gamma^{\mu}u$	V	$\varepsilon = -1$
$\bar{u}\gamma^{5}u$	PS	$\varepsilon = -1$
$\bar{u}\gamma^{\mu}\gamma^{5}u$	PV	$\varepsilon = +1$

2-3- V-A theory

- The idea is to extend the Fermi's theory by looking for Lorentz scalars and pseudo-scalars (to account for the parity violation observed experimentally). Therefore one looks for a transition matrix element of the form :

$$T \propto G \sum_{\substack{i=s,v,\\ T,ps,pv}} \left(\bar{u}_{\nu_e}\theta_i^{\alpha}(c_i - c'_i\gamma_5)u_e\right)\left(\bar{u}_n\theta_{i\alpha}u_p\right)$$

with the operators taken as generic bilinear covariants.

- In details:

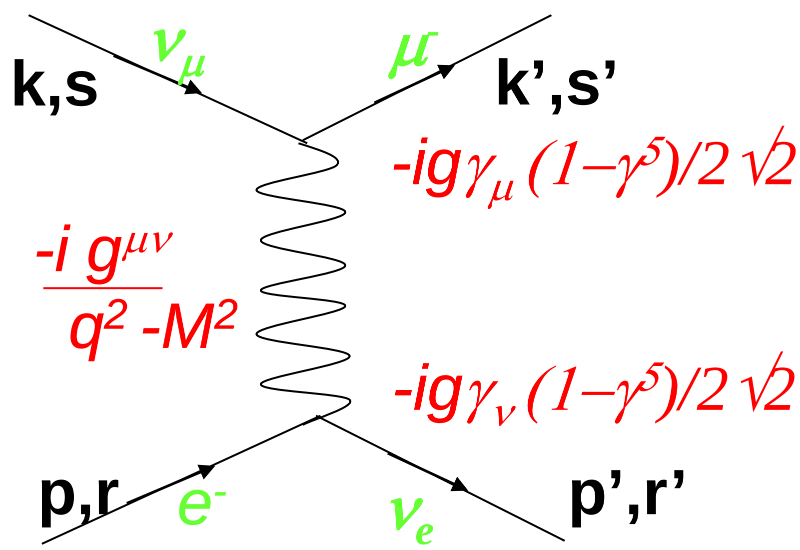
$$T \propto \frac{G}{\sqrt{2}}\Big[\underbrace{\left(\bar{u}_{\nu_e}\gamma^{\alpha}(1-\gamma_5)u_e\right)}_{V-A}\underbrace{\left(\bar{u}_n\gamma_{\alpha}(c_v - c_a\gamma_5)u_p\right)}_{V-A}\Big]$$

$$\frac{c_a}{c_v} = 1,25$$

3-1- Leptonic processes

- What is the link between the Fermi approach and the general Feynman diagram formalism?
- Let’s consider the leptonic process :

$$\nu_\mu(k) + e^-(p) \rightarrow \mu^-(k') + \nu_e(p')$$

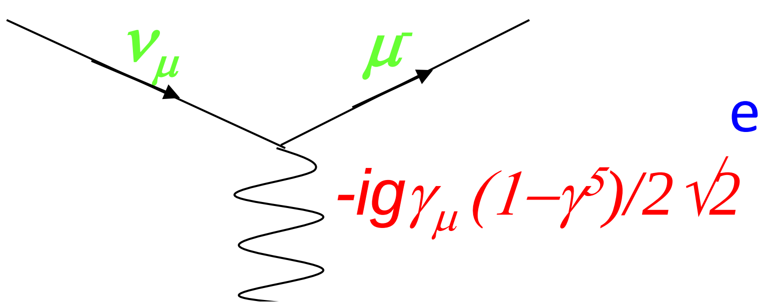


3-1- Leptonic processes

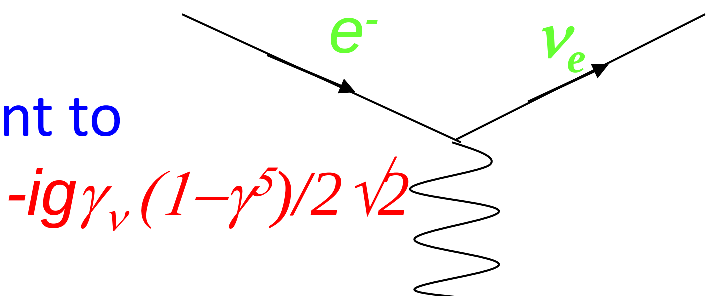
- Orders of magnitude :

$$\sigma = \frac{G_F^2 s}{\pi} = 10^{-41} E_\nu(\mathrm{GeV})\ \ \mathrm{cm}^2$$

- Remark : in the leptonic sector holds a coupling universality :

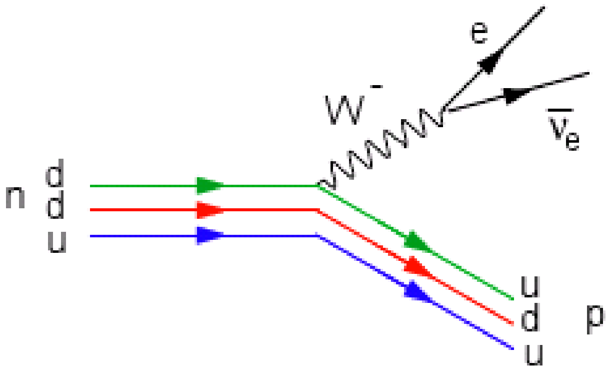


equivalent to

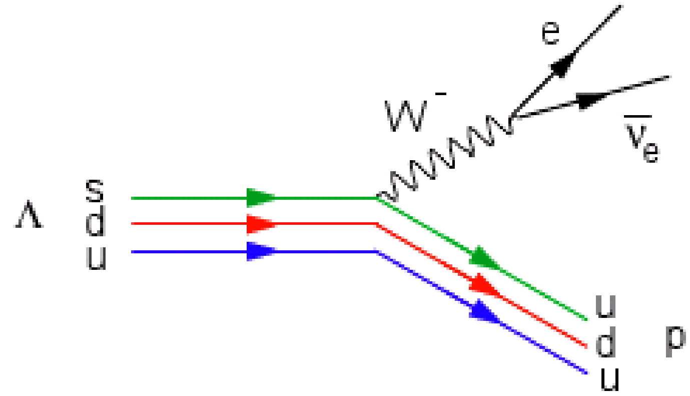


3-2- Quarks and flavor mixing

- Experimentally two types of transition are observed :



$\Delta S=0$



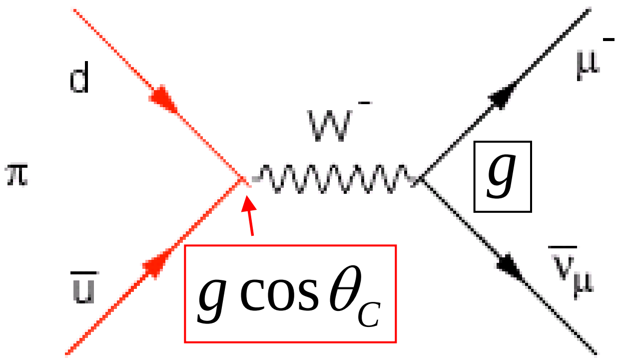
$\Delta S=1$

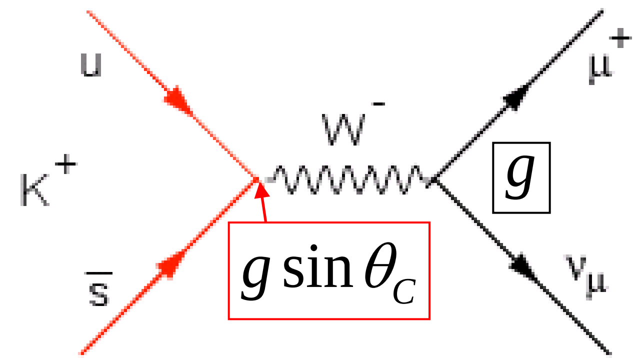
- The transition rates of the 2nd ones are ~20 times lower than the ΔS=0 ones.
- Explanation proposed (Cabbibo, 1963) : the quarks weak doublets involve linear combinations of the quarks carrying same quantum numbers :

$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d' \end{pmatrix}\begin{pmatrix} c \\ s' \end{pmatrix} \text{ with } \begin{cases} d' = \cos\theta_C d + \sin\theta_C s \\ s' = -\sin\theta_C d + \cos\theta_C s \end{cases}$$

3-2- Quarks and flavor mixing

- The coupling strength are modified according to :





- In that approximation transitions implying a strangeness change are proportional to $T_{fi} \propto G_F \sin\theta_C$ while the ones implying no change in the strangeness are $T_{fi} \propto G_F \cos\theta_C$

$n \rightarrow p + e^- + \bar{\nu}_e$	$d \rightarrow u$	$G_F^2 \cos^2\theta_C$
$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$	$u \rightarrow d$	$G_F^2 \cos^2\theta_C$
$K^+ \rightarrow \pi^0 + e^- + \bar{\nu}_e$	$s \rightarrow u$	$G_F^2 \sin^2\theta_C$
$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$	—	G_F^2

3-2- Quarks and flavor mixing

- The ratio between complementary processes allow to compute the value of the Cabbibo angle:

$$\frac{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)} = \tan^2\theta_C = 0.05 \Rightarrow \theta_C = 13°$$

- This model extends to 6 quarks with the introduction of the C.K.M. matrix (Cabbibo-Kobayashi-Maskawa) :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = M \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$ with $$M = \begin{pmatrix} c_1 & c_3 s_1 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + c_3 s_2 e^{i\delta} \\ s_1 s_2 & -c_1 c_3 s_2 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

- Experimentally : $M = \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ -0.22 & 0.97 & 0.04 \\ 0.004 & -0.04 & 0.99 \end{pmatrix}$

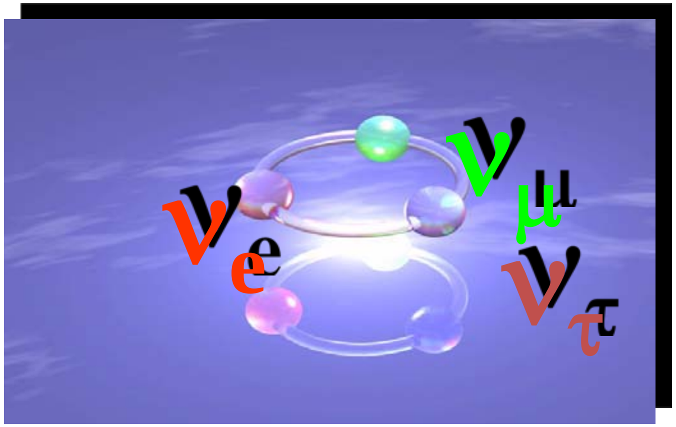
- N.B. M∈₵ because of CP violation

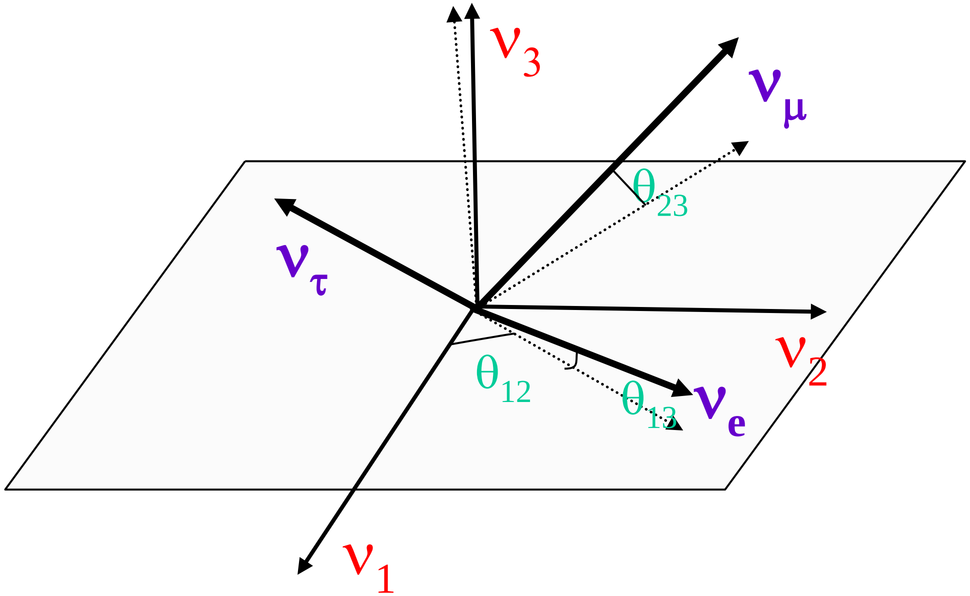
3-3- Neutrino properties

- What about leptonic sector? The CKM matrix is being extensively measured and is a quasi-diagonal matrix.

- Same procedure is believed to apply in the leptonic sector, responsible for the neutrino oscillations. Mass eigenstates differ from interaction eigenstates implying the existence of an unitary matrix (PMNS : Pontecorvo Maki Nakagawa Sakata)

$$\upsilon_\alpha = \sum_i U_{\alpha i} \upsilon_i$$





3-3- Neutrino properties

- In the case of non-vanishing neutrino masses there is a non-zero probability to observe a flavor transition from the source point to the detection point :

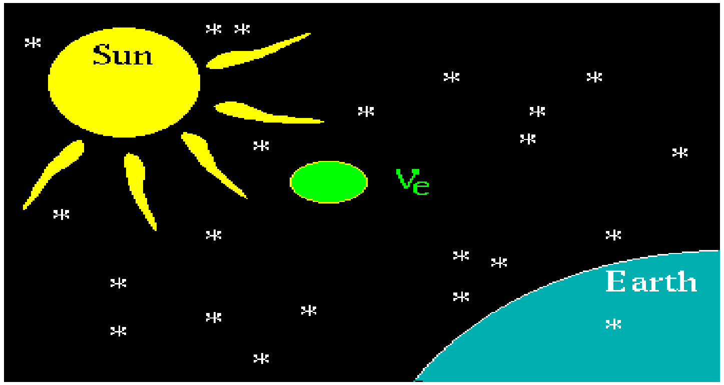
$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\,\mathrm{Re} \sum_{i<j} U_{\beta i}^{*} U_{\beta j} U_{\alpha i} U_{\alpha j}^{*} \sin^2 \frac{\Delta m_{ij}^2 L}{4E} + 2\,\mathrm{Im} \sum_{i<j} U_{\beta i}^{*} U_{\beta j} U_{\alpha i} U_{\alpha j}^{*} \sin^2 \frac{\Delta m_{ij}^2 L}{2E}$$

- Experimentally one can probe the different parts of the PMNS matrix using different neutrino sources :

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

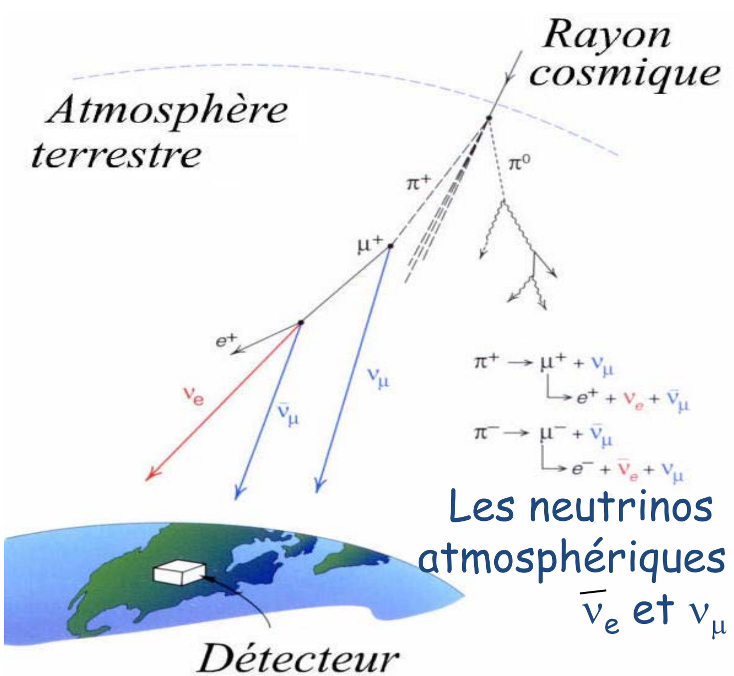
atmospheric ν   reactors   solar ν

3-3- Neutrino properties



40 milliards / cm^2 / sec (some MeV)

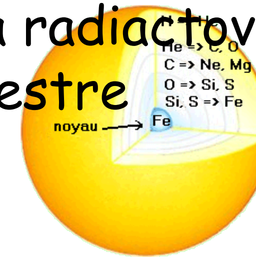
Les neutrinos du Big-Bang :
- les trois sortes de neutrinos
- environ 330 ν / cm^3
  (soit un milliard de fois plus que de protons)
- environ 0,0004 eV



Les neutrinos atmosphériques $\bar{\nu}_e$ et ν_μ

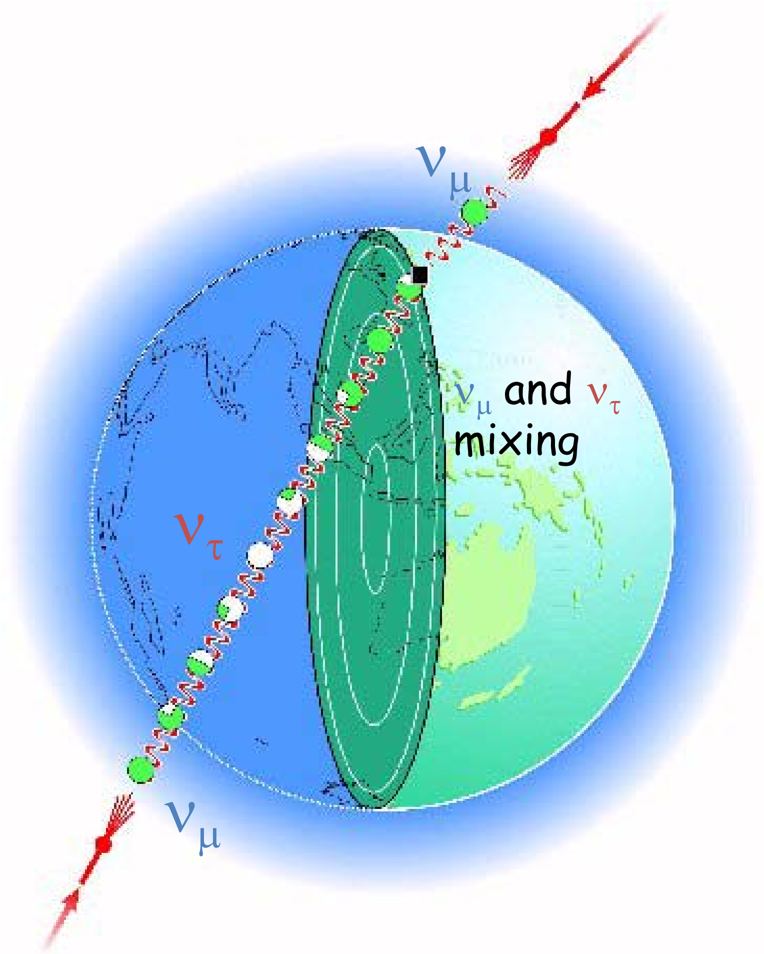
Les neutrinos des centrales nucléaires ν_e

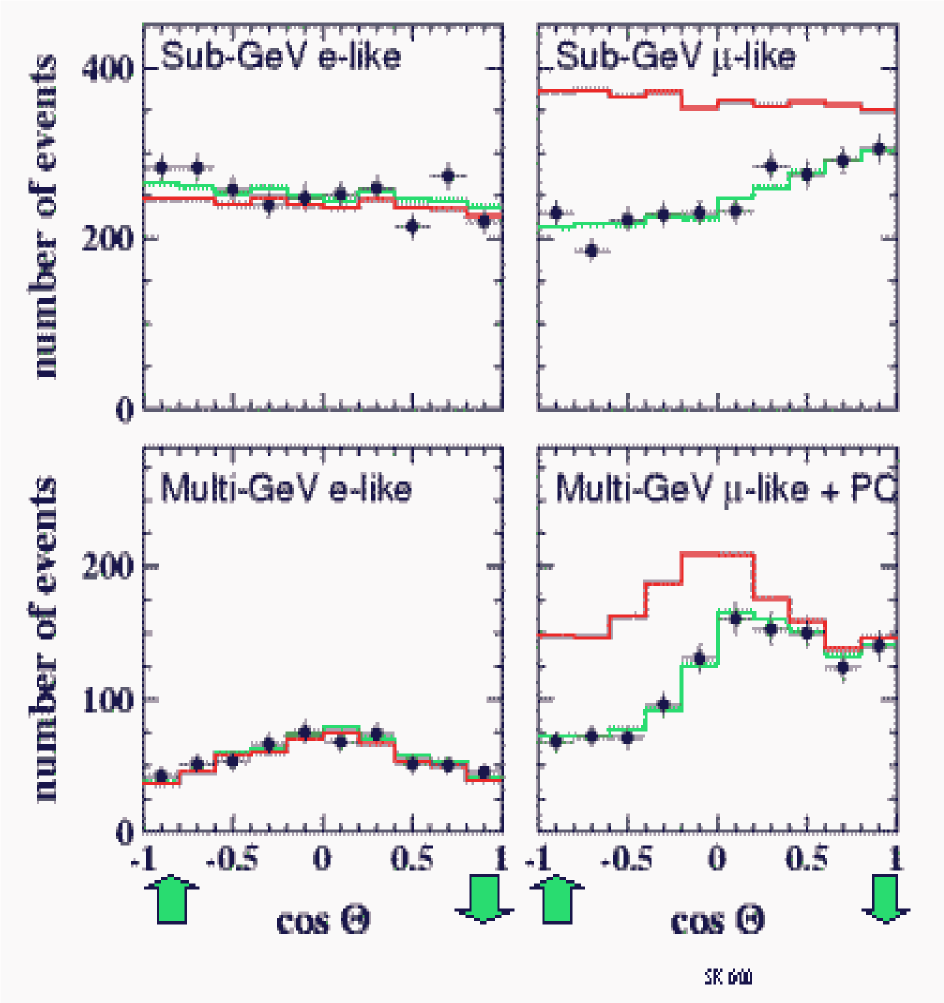
Les neutrinos de la radiactovité terrestre



3-3- Neutrino properties

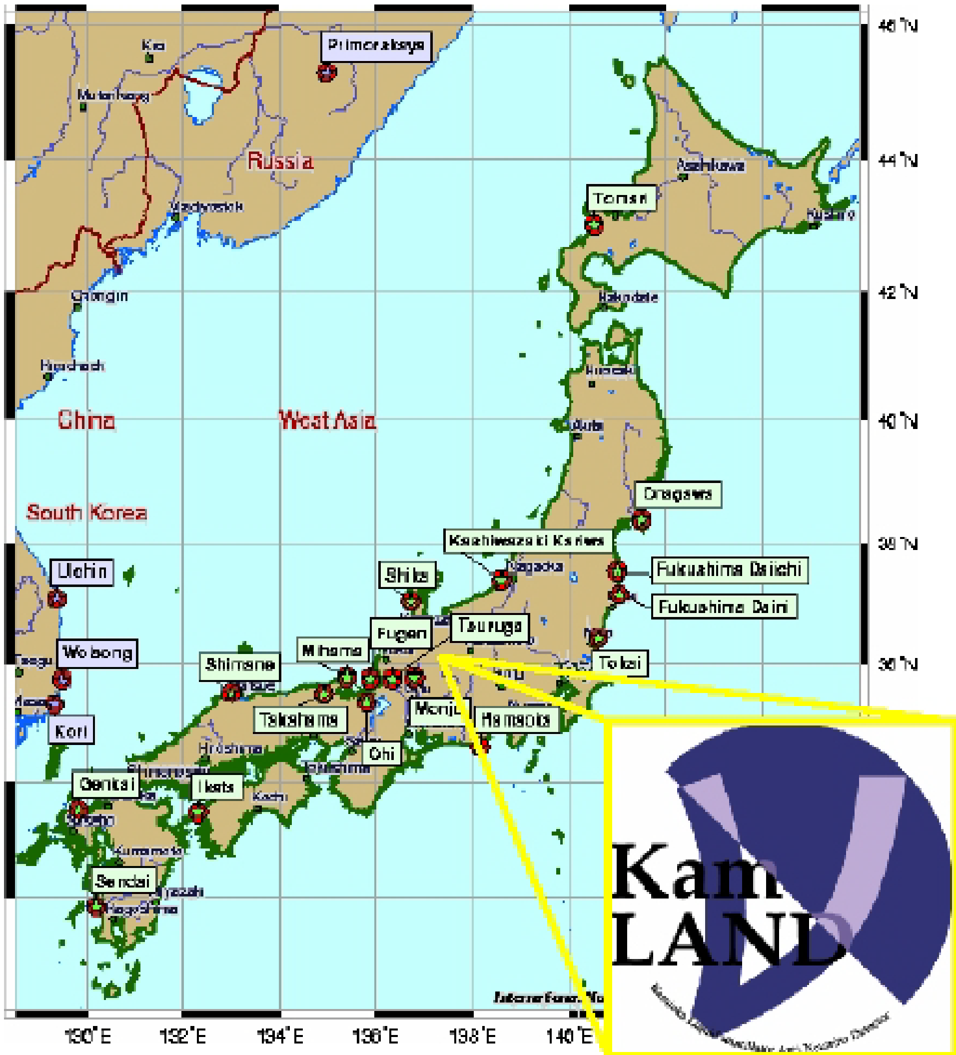
- Neutrino oscillations : observations / explanations

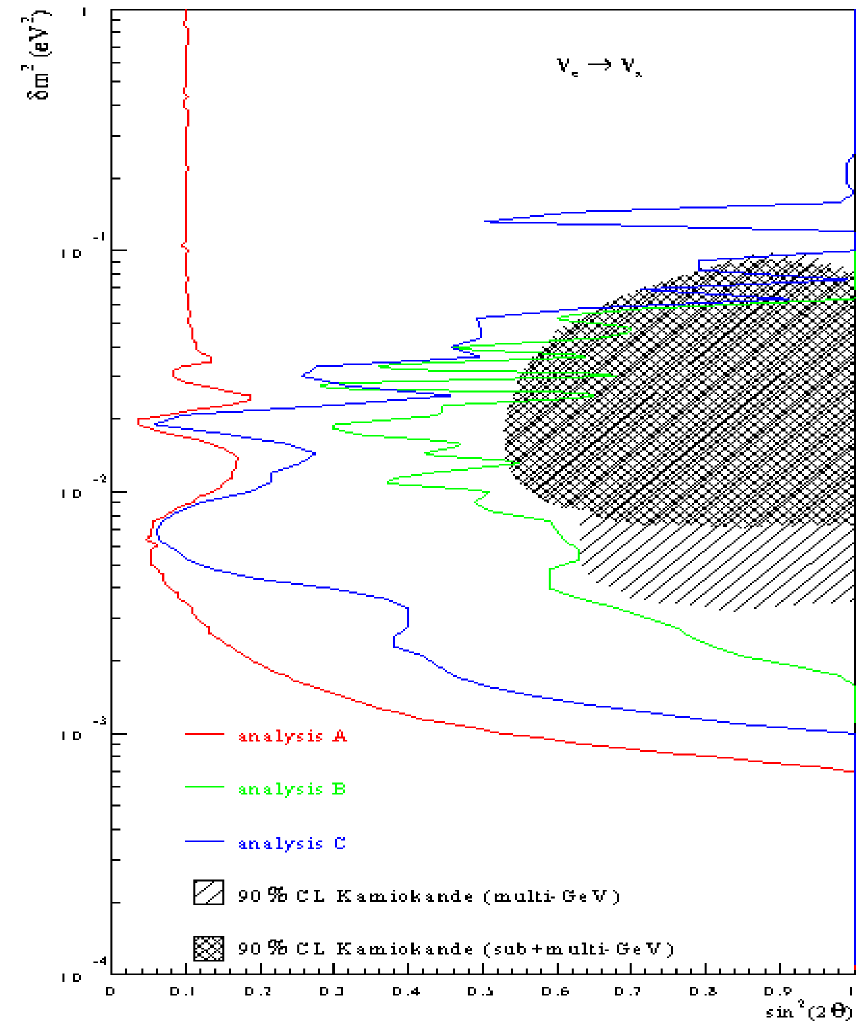




3-3- Neutrino properties

- Neutrino oscillations on nuclear reactors : KamLAND and Chooz

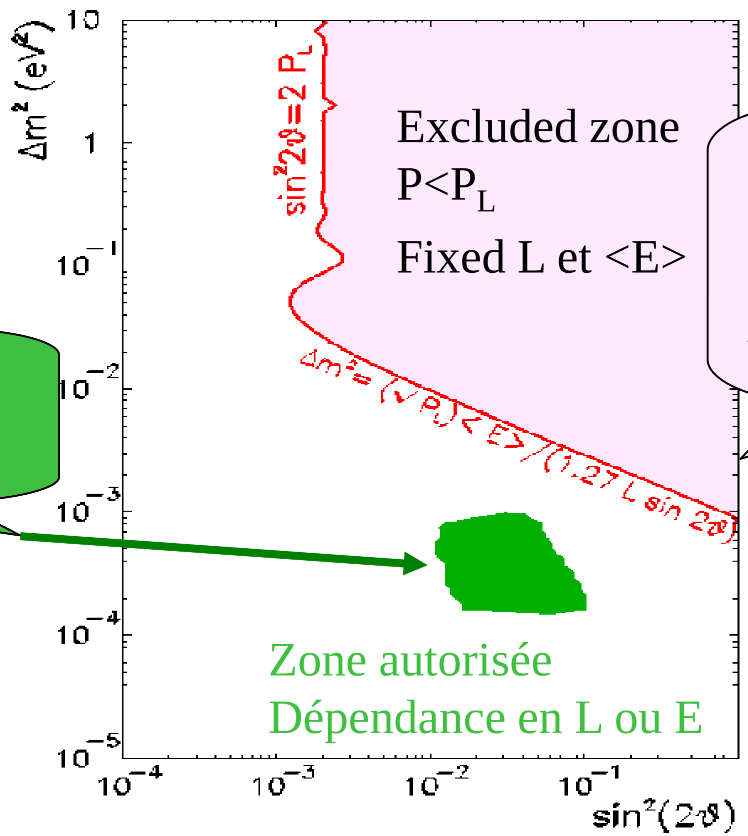




3-3- Neutrino properties

- Neutrino oscillations analysis in the parameter space

Observed signal
Allowed parameter zone

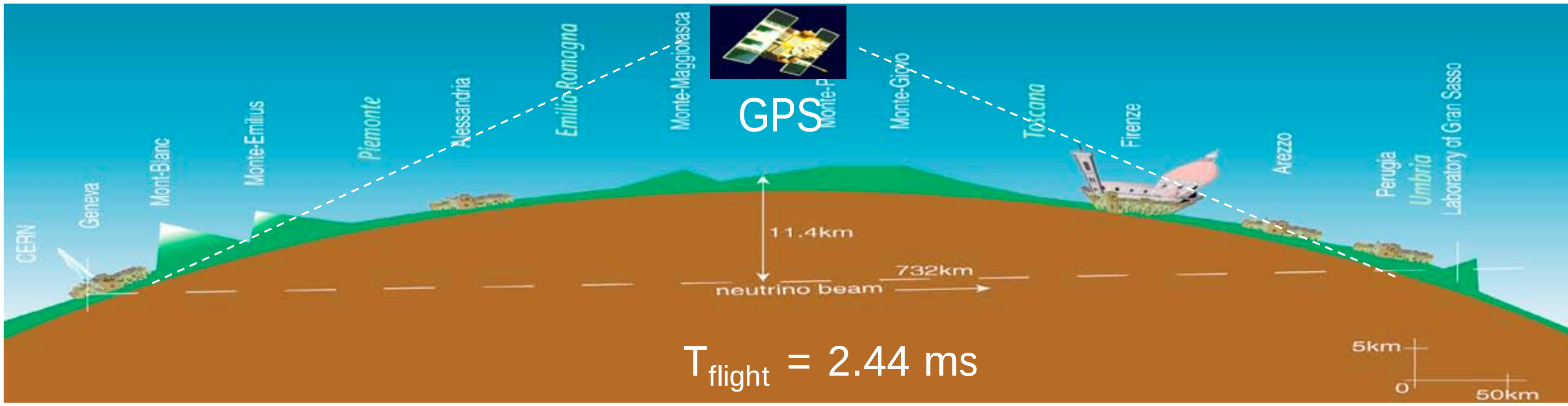


Negative research :
excluded zone in
Δm^2 and $\sin^2(2\theta)$

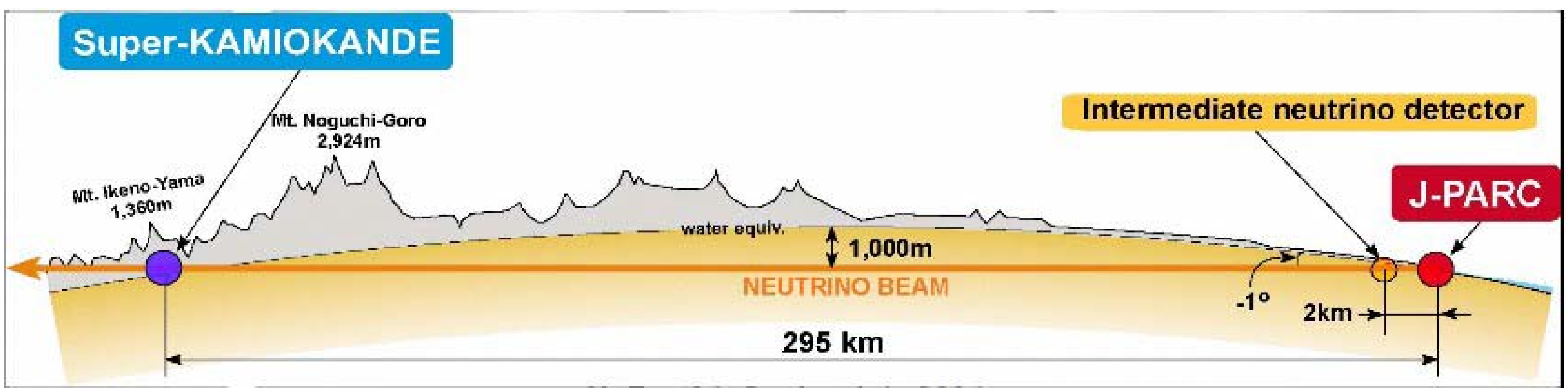
3-3- Neutrino properties

- Neutrino oscillations on accelerators :

OPERA



$\nu_\alpha \longleftrightarrow \nu_\beta$



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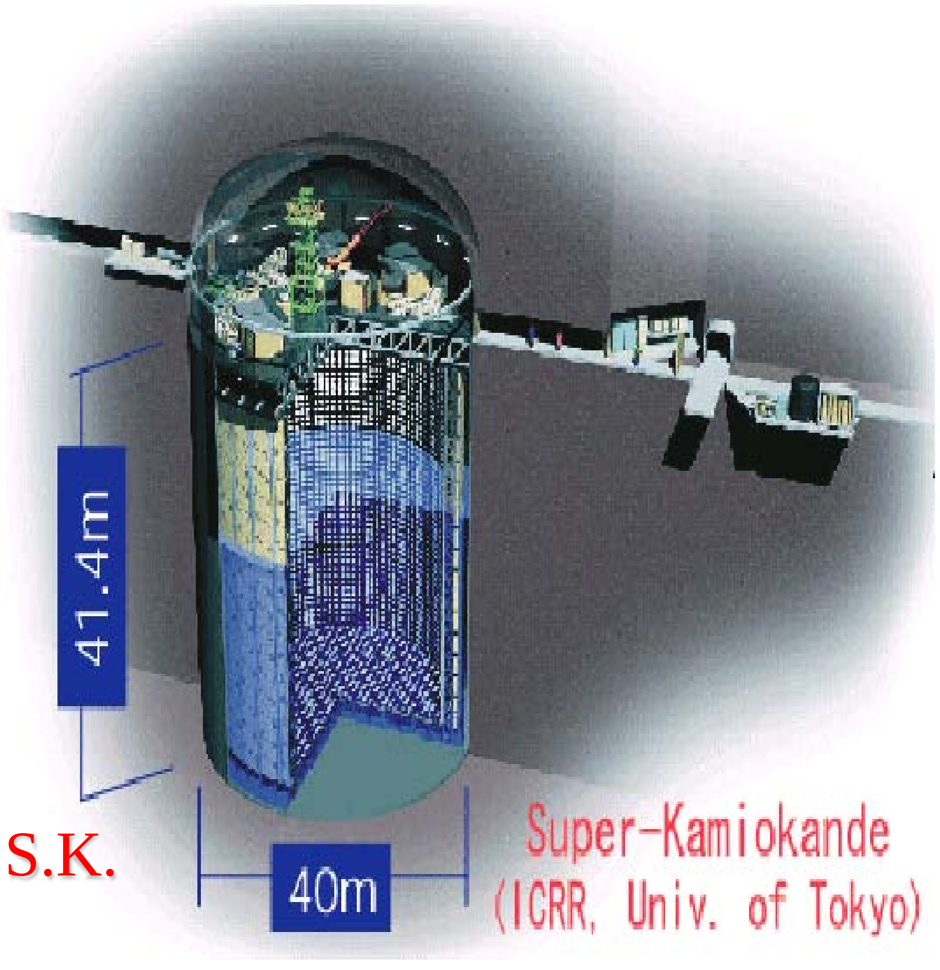
3-3- Neutrino properties

- Neutrino oscillations on accelerators :



3-3- Neutrino properties

Conclusions on neutrinos :

- Weak interactions only
  - => difficult detection (low cross-sections)
  - => symmetry breaking (C, P, CP?)
- Potentially massive but with a low mass value (why?)
- Mixing exists in the leptonic sector as well (but the unitary matrix is almost bimaximal, why?). 1 parameter still unknown…
- Many open questions (cosmological role? Symmetry breaking mechanisms? Majorana particles? More than 3 neutrinos?...)