Master Sciences de la Matière **ENS Lyon**
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Travaux Dirigés de Physique des Particules

QED : e^+e^- Bhabha scattering
Feynman Diagrams, Feynman Rules, cross-section calculation

Reminder : QED Feynman Rules
These rules, originated from Quantum Field Theory, are applied on Feynman diagrams in order to evaluate the transition matrix element. For the external lines of a diagram :

- Each incoming spin $\frac{1}{2}$ particle is associated to : $u(p,s)$
- Each outgoing spin $\frac{1}{2}$ particle is associated to : $\bar{u}(p,s)$
- Each incoming spin $\frac{1}{2}$ antiparticle is associated to : $\bar{v}(p,s)$
- Each outgoing spin $\frac{1}{2}$ antiparticle is associated to : $v(p,s)$
- Each incoming photon is associated to : $\varepsilon_\mu(p,\lambda)$
- Each outgoing photon is associated to : $\varepsilon^*_\mu(p,\lambda)$

For the internal lines of a diagram :

- Photon propagator : $\frac{-ig^{\mu\nu}}{q^2}$
- Fermion propagator : $\frac{i\gamma^\mu p_\mu + m}{q^2 - m^2}$
- Massive boson propagator : $\frac{i}{q^2 - m^2}$

For a vertex between a photon and to charge e fermions : $-ie\gamma^\mu$

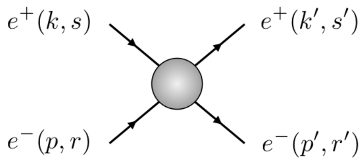
In general there is more than one Feynman diagram contributing to a process and sometimes identical particles appear in the initial or final states.

- Write all topological inequivalent diagrams at any order in perturbation theory.
- The overall sign of a given diagram is not observable, but diagrams that differ in the exchange of two identical fermions in the initial or final state, or a fermion-antifermion in the initial or final state, respectively, should come with opposite signs on account of Fermi statistics.

Problem : Bhabha scattering : $e^+e^- \longrightarrow e^+e^-$

1 / Draw the Feynman diagrams contributing to this scattering. In this problem, we restraint our study to the dominant processes at low energy.

2 / The kinetic variables are defined in the following figure. Give the Mandelstam variables $s = (k+p)^2$, $t = (k-k')^2$ and $u = (k-p')^2$ in the ultra relativist limit.



3 / Use Feynman rules to calculate the transition matrix element T, ans it's conjugate T^*

4 / Show that $|T|^2$ is the sum of three contributions : scattering (T_t), annihilation (T_s) and interference.

5 / Show that the contribution of the annihilation term averaged on the initial polarizations and summed on the final polarizations is:

$$\overline{|T_s|^2} = \frac{e^4}{4q^4}\,\mathrm{Tr}((\not{k} \;\; - m_e)\gamma_\mu(\not{p} \;\; + m_e)\gamma_\nu)\,\mathrm{Tr}((\not{p}' \;\; + m_e)\gamma^\mu(\not{k}' \;\; - m_e)\gamma\nu)$$

6 / Calculate $\overline{|T_s|^2}$ in the ultra-relativist limit.
Reminder : some properties of the γ matrices :

$$\begin{aligned}
\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta) &= 4(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}) \\
\gamma_\alpha\gamma_\mu\gamma_\nu\gamma^\alpha &= 4g_{\mu\nu} \\
\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\rho\gamma^\alpha &= -2\gamma_\rho\gamma_\nu\gamma_\mu
\end{aligned}$$

7 / Repeat step 5 and 6 to calculate the contributions of the scattering and the interference terms.

8 / Show that the Bhabha differential cross-section is :

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{u^2+t^2}{s^2}\right)$$

9 / Let θ be the scattering angle in the center of mass frame. Show that in this frame :

$$t = -\frac{s}{2}(1 - \cos\theta) \quad \text{and} \quad u = -\frac{s}{2}(1 + \cos\theta)$$

Give the Bhabha cross section in the center of mass frame.

10 / How to calculate the Møller scattering cross-section ($e^-e^- \longrightarrow e^-e^-$) from the previous results ? Show that

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left(\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}\right)$$