Master of Physics **ENS Lyon**
Year 2009-2010

Particle Physics

Kinematics

1 Target vs. centre of mass

Consider the reaction

$$\bar{p} + p \to \overline{\Lambda} + \Lambda \ ,$$

with masses $m = 0.938\,\mathrm{GeV}/c^2$ and $M = 1.116\,\mathrm{GeV}/c^2$.

1/ For given $s = (\tilde{p}_p + \tilde{p}_{\bar{p}})^2$, calculate the antiproton momentum q in the proton frame and q^* in the c.o.m. frame. What are the numerical values at threshold? What is the relation between q and q^* for very large s?

2/ For $s = 15\,\mathrm{GeV}^2$, what is the average distance between the production and the decay of a Λ produced forward, in both frames. The lifetime of Λ is $\tau = 2.63 \times 10^{-10}\,\mathrm{s}$.

3/ Show that in the target frame, the hyperons are emitted in a cone. Calculate the maximal angle as a function of s and of the masses.

4/ Give the general expression of the momentum of particles a and b, with respective masses m_a and m_b, entering the reaction $a + b \to \cdots$, in both the target frame attached to b and the c.o.m. frame.

2 Pion decay in flight

The neutral pion, of mass $m = 0.1349\,\mathrm{GeV}/c^2$ decays into two photons, $\pi^0 \to \gamma + \gamma$. One now considers a beam of neutral pions with energy E (including the rest mass energy).

1/ Estimate the momentum of these pions. What are the parameters of the Lorentz transformation from the laboratory to the c.o.m.?

2/ A photon is emitted with an energy $mc^2/2$ and angle ϑ^* in the frame of the pion. Calculate its energy and momentum in the laboratory. Calculate the relation between ϑ^* and the angle ϑ in the lab.,

3/ The angular distribution is isotropic in the pion frame, that is to say

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega^*} = \frac{N_0}{4\pi} \quad \text{or} \quad \frac{\mathrm{d}N}{\mathrm{d}\cos\vartheta^*} = \frac{N_0}{2} \ .$$

Calculate the angular distribution $\mathrm{d}N/\mathrm{d}\cos\vartheta$ in the laboratory.

3 Kinematics of $\boldsymbol{a} + \boldsymbol{b} \longrightarrow \mathbf{1} + \mathbf{2} + \cdots + \boldsymbol{n}$

The cross-section of a process $a + b \to 1 + 2 + \cdots + n$ is given by:

$$\sigma_{if} = \frac{1}{2E_a\, 2E_b\, |\boldsymbol{v}_a - \boldsymbol{v}_b|} \int (2\pi)^4\, \delta^{(4)}(p_f - p_i)\, \overline{|T_{fi}|^2} \prod_{k=n}^{k=1} \frac{d^3 p_k}{(2\pi)^3\, 2E_k}$$

where $T_{fi} = \langle f|T|i\rangle$ is the **transition matrix element** between the initial and the final states and contains the information about the dynamics of the process. The other factors are due to kinematics:

- $(2E_a\, 2E_b\, |\boldsymbol{v}_a - \boldsymbol{v}_b|)$ is the **flux factor**,
- $dQ_n = (2\pi)^4\, \delta^{(4)}(p_f - p_i) \prod_{k=n}^{k=1} d^3p_k/((2\pi)^3\, 2E_k)$ describes the **phase-space volume** available in the final state.

1/ The above relation is written assuming that the states are normalised to $2E$ particles per unit of volume, *i.e.*, a state $|p, \lambda\rangle$ with wave function $\langle x|p, \lambda\rangle = u_\lambda \exp[i(E \cdot t - \boldsymbol{p} \cdot \boldsymbol{x})]$ fulfils

$$\langle p', \lambda'|p, \lambda\rangle = 2E\, \delta_{\lambda\lambda'}\, (2\pi)^3\, \delta^3(\boldsymbol{p} - \boldsymbol{p}')$$

With this normalisation taken into account, give the physical meaning of the flux factor.

2/ Calculate the flux factor in the c.o.m. frame of $a + b$, as a function of the 3-momentum and of the Mandelstam variable $s = (p_a + p_b)^2$.

3/ Calculate the two-body phase-space for, say, $a + b \to 1 + 2$, and show that

$$dQ_2 = \frac{1}{16\,\pi^2} \frac{|\boldsymbol{p}_1|^3\, d\Omega}{|\boldsymbol{p}_1|^2\, E_T - \boldsymbol{p}_1 \cdot \boldsymbol{p}_T\, E_1}$$

where E_T denotes the total energy in the initial state, $\boldsymbol{p}_T$ the total three-momentum and $d\Omega$ the elementary solid angle associated to $\boldsymbol{p}_1$.

4/ Calculate the differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$ in the c.o.m. frame. Simplify the expression in the limit where the masses are neglected.