

# *Chapter 1*

## *Particles...*

*How do we classify them?*

*How do they interact?*

*How do we detect them?*

# Outline/Plan

## 1. Fundamental particles

1. Leptons
2. Quarks
3. Hadrons

## 2. Hadron spectroscopy

1. Isospin symmetry
2. Basics of group theory.  
The  $SU(N)$  group.
3. The quark model

## 3. Fundamental interactions

1. Range and propagators
2. Electro-weak interaction
3. Strong interaction

## 1. Particules fondamentales

1. Leptons
2. Quarks
3. Hadrons

## 2. Spectroscopie hadronique

1. La symétrie d'isospin
2. Rappel de théorie des groupes.  
Le groupe  $SU(N)$ .
3. Le modèle des quarks.

## 3. Interactions fondamentales

1. Portée d'une interaction et  
propagateurs.
2. Interaction électro-faible.
3. Interaction forte.

# 1- Fundamental particles

General features:

- Fundamental particles can not be separated into smaller components (elementary particles such as: electron, photon, quarks...)
- Some particles are composite ones (protons and neutrons are composed of 3 quarks, pions of 1 quark and 1 anti-quark...)
- There are 2 ways of classifying the particles:

1. Following the spin-statistics

Fermions (1/2 integer spin, Fermi-Dirac statistics)

Vs

Bosons (integer spin, Bose-Einstein statistics)

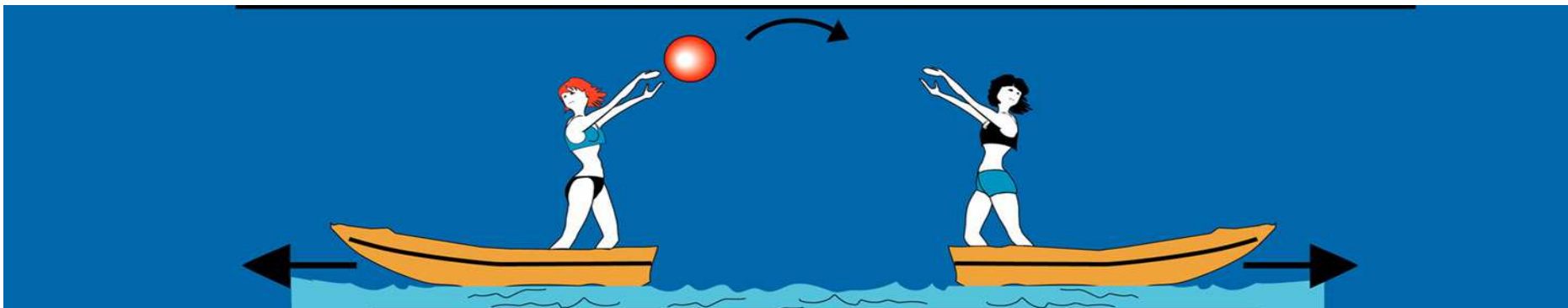
2. Following the interaction(s) they are sensitive to...

# 1- Fundamental particles

There are 4 fundamental interactions:

- **Strong** interaction (e.g. nuclei structure)
- **Electromagnetic** interaction (e.g. atomic physics, light,  $\mu$ -wave...)
- **Weak** interaction (e.g.  $\beta$  radioactivity phenomena)
- **Gravity** (neglected at energy scales well below  $10^{19}$ GeV)

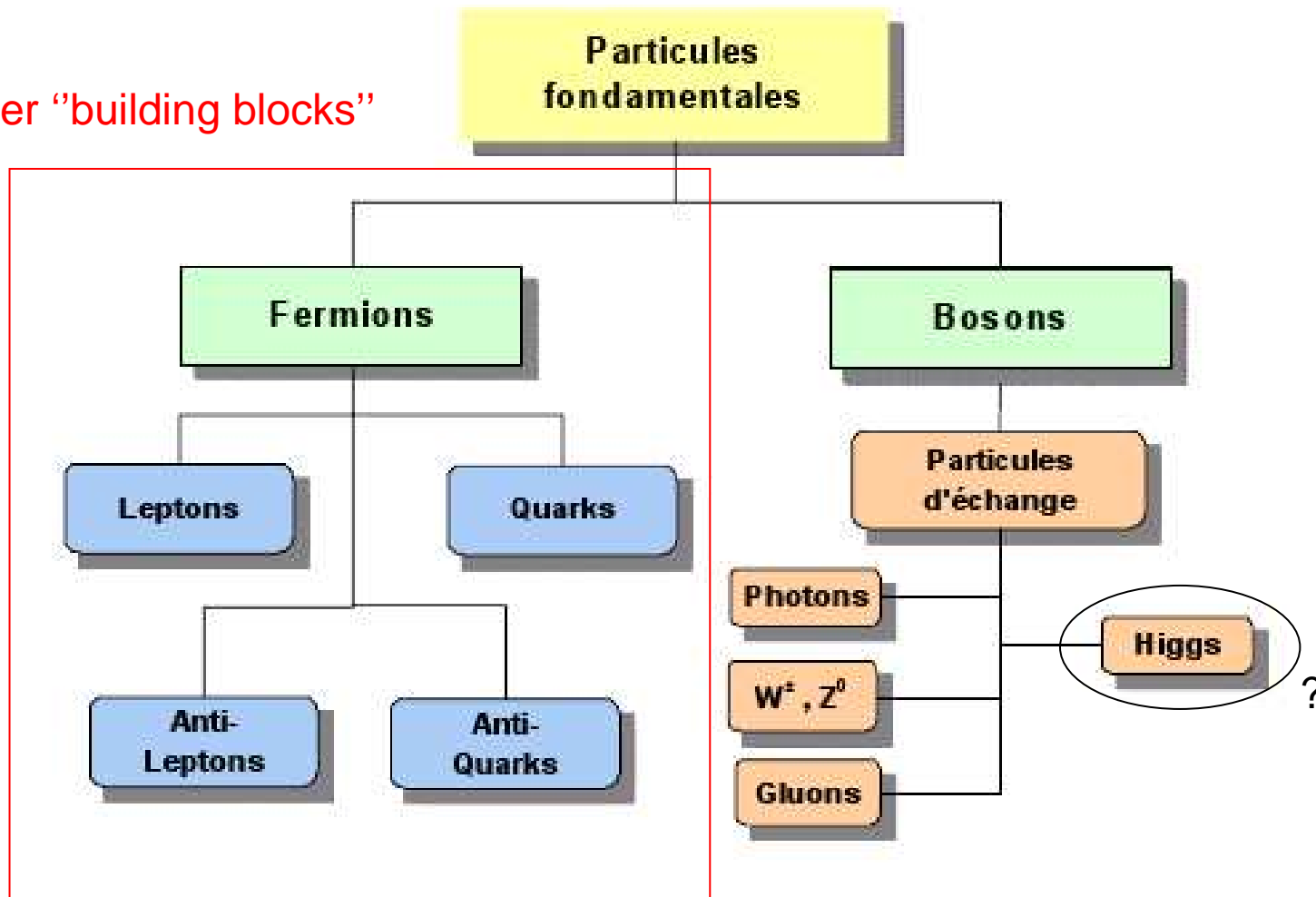
In quantum field theory any interaction is modeled by an intermediate particle exchange (**gauge bosons**).



# 1- Fundamental particles

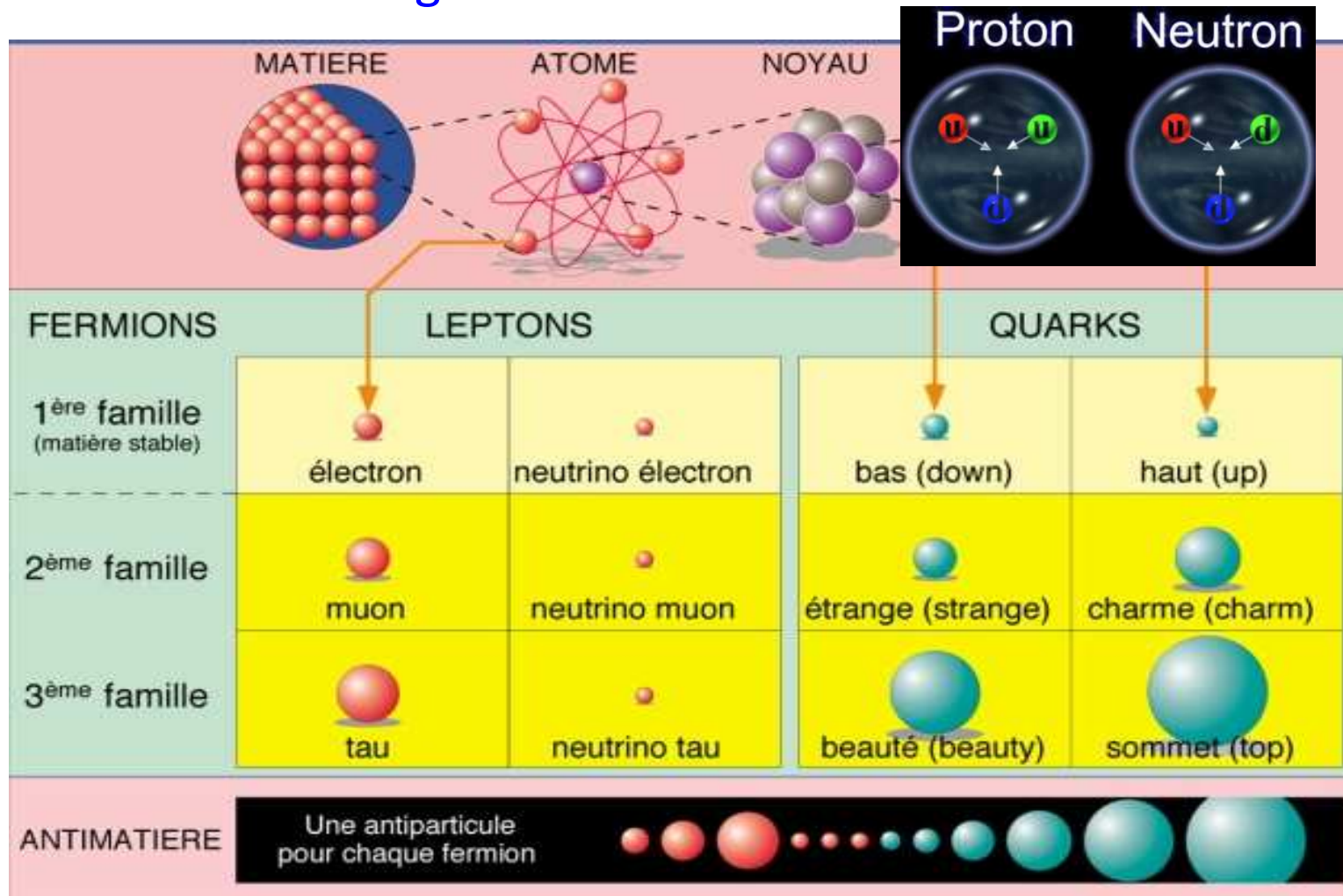
A simplified scheme

Matter "building blocks"



# 1.1 Leptons and quarks

They are (up-to-now) the most elementary particles known and constitute the building blocks of atoms



# 1.1 Leptons

Leptons:

- Are insensitive to strong interaction
- Carry integer electric charges ( $n \times 1.610^{-19} \text{C}$  with  $n \in \mathbb{Z}$ )
- Carry a “weak” charge ie can be associated in weak interaction doublets
- Are organized into 3 families : electron, muon, tau

| Leptons (spin $\frac{1}{2}$ ) |  |  |  |
|-------------------------------|--|--|--|
| $Q = 0$                       | $\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$ | $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$ | $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ |
| $Q = -1$                      | $\begin{pmatrix} e \end{pmatrix}$                            |  |  |

- Muons and taus are “heavy” and unstable copies of electrons

# 1.1 Leptons

**Leptonic number:** global symmetry associated to leptons implying that 3 numbers are conserved additively in the interactions:

- $L_e = +1$  ( $e^-$  and  $\nu_e$ ) /  $L_e = -1$  ( $e^+$  and  $\bar{\nu}_e$ ) /  $L_e = 0$  for others
- $L_\mu = +1$  ( $\mu^-$  and  $\nu_\mu$ ) /  $L_\mu = -1$  ( $\mu^+$  and  $\bar{\nu}_\mu$ ) /  $L_\mu = 0$  for others
- $L_\tau = +1$  ( $\tau^-$  and  $\nu_\tau$ ) /  $L_\tau = -1$  ( $\tau^+$  and  $\bar{\nu}_\tau$ ) /  $L_\tau = 0$  for others
- Reactions example:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (L_\mu: 0 = -1 + (+1))$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (L_e: 0 = +1 + (-1) / L_\mu: +1 = +1)$$



# 1.1 Leptons

Neutrinos are only sensitive to weak interactions and have a fixed

helicity (operator :  $\lambda = \frac{\vec{S} \cdot \vec{p}}{p}$  )



# 1.1 Leptons

## Leptons summary

| Leptons                          |                           |                                |     |                        |   |               |
|----------------------------------|---------------------------|--------------------------------|-----|------------------------|---|---------------|
| $S, C, \tilde{B}, T, I, I_3 = 0$ |                           |                                |     |                        |   |               |
|                                  | $M$ (MeV)                 | $\tau$                         | $Q$ | $(L_e, L_\mu, L_\tau)$ | $(I^W, I_3^W)_{R,L}$                      | $J^{PC}$      |
| $e$                              | 0.51099892(4)             | $> 4.6 \times 10^{26}$ ans     | -1  | (1, 0, 0)              | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | $\frac{1}{2}$ |
| $\nu_e$                          | $< 3 \times 10^{-6}$      | $> 300 m_\nu$ s/eV             | 0   | (1, 0, 0)              | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | $\frac{1}{2}$ |
| $\mu$                            | 105.658369(9)             | $2.197030(4) \times 10^{-6}$ s | -1  | (0, 1, 0)              | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | $\frac{1}{2}$ |
| $\nu_\mu$                        | $< 0.19$                  | $> 15.4 m_\nu$ s/eV            | 0   | (0, 1, 0)              | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | $\frac{1}{2}$ |
| $\tau$                           | $1776.99^{(+29)}_{(-26)}$ | $290.6(11) \times 10^{-15}$ s  | -1  | (0, 0, 1)              | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | $\frac{1}{2}$ |
| $\nu_\tau$                       | $< 18.2$                  | —                              | 0   | (0, 0, 1)              | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | $\frac{1}{2}$ |

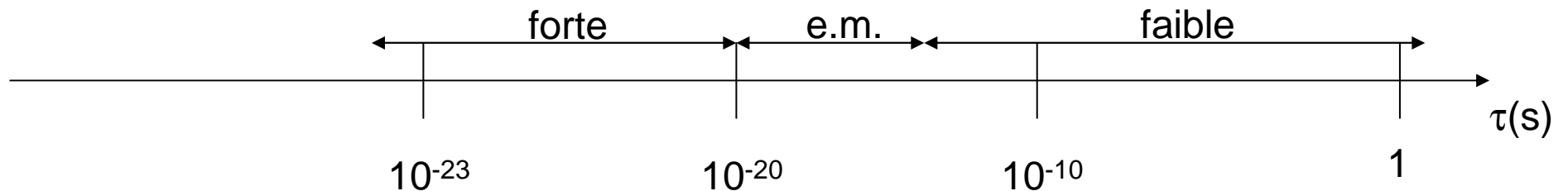
### Notations:

- $I^W$  and  $I_3^W$  are related to the weak isospin
- $J^{PC} = \text{Spin}^{\text{Parity C-Parity}}$

# 1.1 Leptons

What about **stability** and **lifetime**? Almost all particles (but e.g. electrons, protons) are unstable and decay with a time which depends on the **type of interaction** and the **available phase space**

Hierarchy:



Width (energy scale):

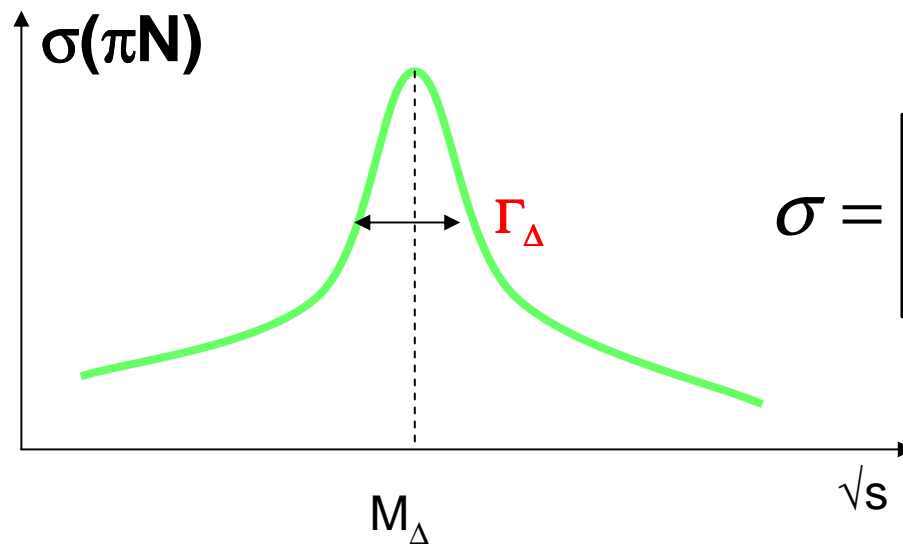
$$\Delta M = \Gamma = \frac{1}{\tau} \quad (\text{MeV})$$

# 1.1 Leptons

Strong decays and resonances :

$$\Delta \rightarrow \pi N \quad (\Gamma = 115 \text{ MeV})$$

$$\rho \rightarrow \pi\pi \quad (\Gamma = 150 \text{ MeV})$$



$$\sigma = \left| \frac{1}{(s^2 - M_\Delta s) + iM_\Delta \Gamma(s)} \right|^2$$

$$\tau \sim 1/100(\text{MeV}) * \left[ 200(\text{MeV} \cdot \text{fm}) / 3 \cdot 10^8 \cdot 10^{15}(\text{fm} \cdot \text{s}^{-1}) \right]$$

$$\tau \sim 10^{-23} \text{ s}$$

# 1.2 Quarks

Quarks:

- Are **sensitive to strong interaction** (they are the fundamental components of nuclear matter)
- Carry **fractional electric charges** (e.g.  $Q_u = 2/3 \times e$ )
- Carry a **“weak” charge** ie can be associated in weak interaction doublets
- Carry also a **“colored charge”** and are associated in triplets of the strong interaction
- Are organized into **3 families** (as the leptons are, probable link?) which are  $\sim$ identical but for the masses

| Quarks             |  |  |   |
|--------------------|--|--|---|
| $Q = \frac{2}{3}$  | $\begin{pmatrix} u(\text{up}) \\ d(\text{down}) \end{pmatrix}$ | $\begin{pmatrix} c(\text{charm e}) \\ s(\text{étrange}) \end{pmatrix}$ | $\begin{pmatrix} t(\text{top}) \\ b(\text{bottom}) \end{pmatrix}$ |
| $Q = -\frac{1}{3}$ |  |  |   |

# 1.2 Quarks

Quarks are **confined** : they can not be observed in a free state  
(extreme case: quark-gluon plasma)

Global quantum numbers are associated with the quark content of a compound : strangeness (s), charm (c), beauty (b), top (t)...

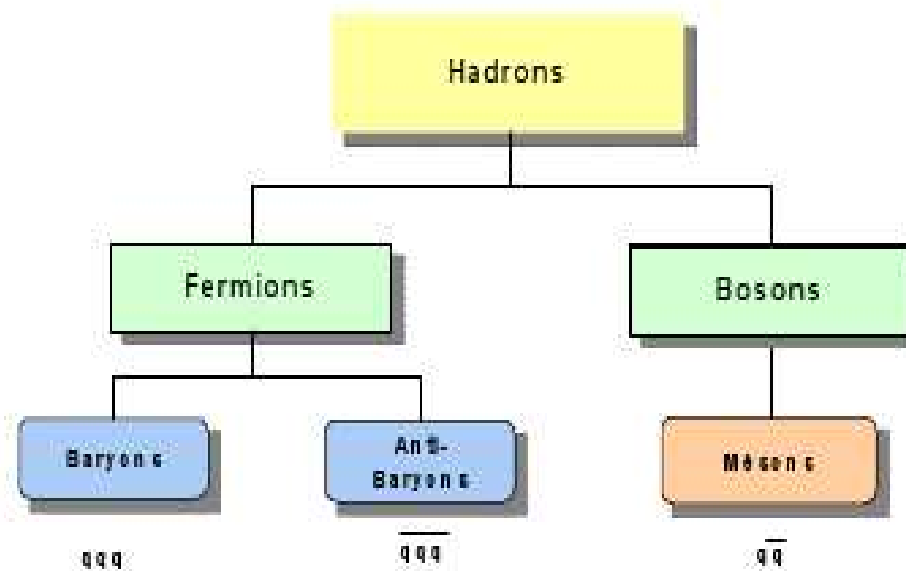
These numbers are conserved in **all but the weak** interactions

| Quarks   |   |                |               |                        |   |               |                 |
|--|---|----------------|---------------|------------------------|---|---------------|-----------------|
| $L_e, L_\mu, L_\tau = 0, I_3 = Q - \frac{1}{2}(B + S + C + \tilde{B} + T)$ |   |                |               |                        |   |               |                 |
|  | $M$ (MeV)   | $Q$            | $B$           | $(S, C, \tilde{B}, T)$ | $(I^W, I_3^W)_{R,L}$                      | $I^G$         | $J^{PC}$        |
| $u$  | 1.5 – 4   | $\frac{2}{3}$  | $\frac{1}{3}$ | (0, 0, 0, 0)           | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
| $d$  | 4 – 8   | $-\frac{1}{3}$ | $\frac{1}{3}$ | (0, 0, 0, 0)           | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
| $s$  | 80 – 130  | $-\frac{1}{3}$ | $\frac{1}{3}$ | (-1, 0, 0, 0)          | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | 0             | $\frac{1}{2}^+$ |
| $c$  | 1150 – 1350   | $\frac{2}{3}$  | $\frac{1}{3}$ | (0, 1, 0, 0)           | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | 0             | $\frac{1}{2}^+$ |
| $b$  | 4100 – 4400 ( $\overline{MS}$ )<br>4600 – 4900 (1S) | $-\frac{1}{3}$ | $\frac{1}{3}$ | (0, 0, -1, 0)          | $(0, 0)_R, (\frac{1}{2}, -\frac{1}{2})_L$ | 0             | $\frac{1}{2}^+$ |
| $t$  | 174.3(51)   | $\frac{2}{3}$  | $\frac{1}{3}$ | (0, 0, 0, 1)           | $(0, 0)_R, (\frac{1}{2}, \frac{1}{2})_L$  | 0             | $\frac{1}{2}^+$ |

# 1.3 Hadrons

Hadrons are the compound particles sensitive to the strong interaction. They are divided into 2 categories:

- **Baryons** : made of 3 quarks ( $q_1q_2q_3$ )
- **Mesons** : made of 1 quark and 1 anti-quark ( $q_1\bar{q}_2$ )



# 1.3 Hadrons

Examples:

| Hadrons                    |               |  |
|----------------------------|---------------|--|
| $p$                        | proton        | — $uud$                                    |
| $n$                        | neutron       | — $udd$                                    |
| $\pi^+, \pi^0, \pi^-$      | pions         | — $u\bar{d}, u\bar{u}+d\bar{d}, \bar{u}d$  |
| $\rho^+, \rho^0, \rho^-$   | mésons $\rho$ | —  |
| $\Lambda$                  | lambda        | — $udc$                                    |
| $K^+, K^0, \bar{K}^0, K^-$ | mésons $K$    | — $u\bar{s}, d\bar{s}, s\bar{d}, \bar{u}s$ |



## 1.3 Hadrons

- Hadrons carry integer electric charge
- They interact weakly
- We associate a global quantum number (**baryonic number**), conserved additively in all reactions and defined as :  
 $B = 1$  for baryons /  $B = -1$  for anti-baryons /  $B = 0$  others

Why do we observe only the baryon/meson combinations only?

Why can we observe such particles as the  $\Delta^{++} = (u\uparrow u\uparrow u\uparrow)$  forbidden by the Fermi statistics?

➔ Because of the **colored charge** : the only allowed (physics) states correspond to **“white” combinations** of quarks and antiquarks.

## 1.3 Hadrons

3 basic colors: **R G B** for the quarks and their “anti”-colors for the anti-quarks :  **$\bar{R}$   $\bar{G}$   $\bar{B}$**

White combinations correspond to :

- **RGB** or  **$\bar{R}\bar{G}\bar{B}$**  in equal proportions
- **$R\bar{R}$   $G\bar{G}$   $B\bar{B}$**  in equal proportions  
(where “proportions” means the proper anti-symmetrization)

Example: “white proton” p (**u u d**)

# 1.3 Hadrons

A more exhaustive list... with only a few examples

| Mésons sans saveurs  |               |                                      |            |                        |       |          |
|--|---------------|--------------------------------------|------------|------------------------|-------|----------|
| $L_e, L_\mu, L_\tau, B, S, C, \tilde{B}, T = 0, I_3 = Q$                                   |               |                                      |            |                        |       |          |
| $I = 1 (\pi, \rho, \omega) : u\bar{d}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), d\bar{u}$  |               |                                      |            |                        |       |          |
| $I = 0 (\eta, \eta', h, h', \omega, \phi, f, h) : c_1(u\bar{u} + d\bar{d}) + c_2 s\bar{s}$ |               |                                      |            |                        |       |          |
|  | $M$ (MeV)     | $\Gamma$ (MeV) ou $\tau$             | $Q$        | $(S, C, \tilde{B}, T)$ | $I^G$ | $J^{PC}$ |
| $\pi^\pm$  | 139.57018(35) | $\tau = 2.60330(5) \times 10^{-8}$ s | $\pm 1$    | (0, 0, 0, 0)           | $1^-$ | $0^-$    |
| $\pi^0$  | 134.9766(6)   | $\tau = 8.4(6)10^{-17}$ s            | 0          | (0, 0, 0, 0)           | $1^-$ | $0^{-+}$ |
| $\eta$   | 547.75(12)    | 0.00129(7)                           | 0          | (0, 0, 0, 0)           | $0^+$ | $0^{-+}$ |
| $\eta'$ (958)  | 957.78(14)    | 0.202(16)                            | 0          | (0, 0, 0, 0)           | $0^+$ | $0^{-+}$ |
| $\rho$ (770)   | 775.8(5)      | 150.3(16)                            | $\pm 1, 0$ | (0, 0, 0, 0)           | $1^+$ | $1^{--}$ |
| $\phi$ (1020)  | 1019.456(20)  | 4.26(5)                              | 0          | (0, 0, 0, 0)           | $0^-$ | $1^{--}$ |
| $\omega$ (782)   | 782.59(11)    | 8.49(8)                              | 0          | (0, 0, 0, 0)           | $0^-$ | $1^{--}$ |

| Mésons beaux ( $\tilde{B} = \pm 1$ )   |            |                                      |            |                        |                     |          |
|--|------------|--------------------------------------|------------|------------------------|---------------------|----------|
| $L_e, L_\mu, L_\tau, B, S, \tilde{B}, T = 0, I_3 = Q - \frac{1}{2}\tilde{B}$ |            |                                      |            |                        |                     |          |
| $B^+ = u\bar{b}, B^0 = d\bar{b}, \tilde{B}^0 = \bar{u}b, B^- = \bar{u}b$     |            |                                      |            |                        |                     |          |
|  | $M$ (MeV)  | $\Gamma$ (MeV) ou $\tau$             | $Q$        | $(S, C, \tilde{B}, T)$ | $I^G$               | $J^{PC}$ |
| $B^\pm$  | 5279.0 (5) | $\tau = 1.671(18) \times 10^{-12}$ s | $\pm 1$    | (0, 0, $\pm 1$ , 0)    | $\frac{1}{2}^{\pm}$ | $0^-$    |
| $B^0, \tilde{B}^0$   | 5279.4 (5) | $\tau = 1.536(14) \times 10^{-12}$ s | 0          | (0, 0, $\pm 1$ , 0)    | $\frac{1}{2}^{\pm}$ | $0^-$    |
| $B^*$  | 5325.0 (6) |                                      | $\pm 1, 0$ | (0, 0, 0, 0)           | $\frac{1}{2}^{\pm}$ | $1^-$    |

# 1.3 Hadrons

A more exhaustive list... with only a few examples

| Mésons étranges   |             |  |            |                        |                 |          |
|---|-------------|--|------------|------------------------|-----------------|----------|
| $L_e, L_\mu, L_\tau, B, C, \tilde{B}, T = 0, I_3 = Q - \frac{1}{2} S$<br>$K^+ = u\bar{s}, K^0 = d\bar{s}, \bar{K}^0 = \bar{d}s, K^- = \bar{u}s$<br>$K^{*+} = u\bar{s}, K^{*0} = d\bar{s}, \bar{K}^{*0} = \bar{d}s, K^{*-} = \bar{u}s$ |             |  |            |                        |                 |          |
|   | $M$ (MeV)   | $\Gamma$ (MeV) ou $\tau$                     | $Q$        | $(S, C, \tilde{B}, T)$ | $I^G$           | $J^{PC}$ |
| $K^\pm$   | 493.677(16) | $\tau = 1.2384(24) \times 10^{-8} \text{ s}$ | $\pm 1$    | $(\pm 1, 0, 0, 0)$     | $\frac{1}{2}^+$ | $0^-$    |
| $K^0, \bar{K}^0$  | 497.648(22) |  | 0          | $(\pm 1, 0, 0, 0)$     | $\frac{1}{2}^+$ | $0^-$    |
| $K_S^0$   |             | $\tau = 0.8958(6) \times 10^{-10} \text{ s}$ | 0          | $(0, 0, 0, 0)$         | $\frac{1}{2}^+$ | $0^-$    |
| $K_L^0$   |             | $\tau = 5.18(4) \times 10^{-8} \text{ s}$    | 0          | $(0, 0, 0, 0)$         | $\frac{1}{2}^+$ | $0^-$    |
| $K^*(892)$  | 891.66(26)  | 50.8(9)                                      | $\pm 1, 0$ | $(\pm 1, 0, 0, 0)$     | $\frac{1}{2}^+$ | $1^-$    |
| $K^{*0}, \bar{K}^{*0}(892)$   | 896.10(27)  | 50.7(6)                                      | 0          | $(\pm 1, 0, 0, 0)$     | $\frac{1}{2}^+$ | $1^-$    |

| Mésons étranges charmés $S = C = \pm 1$   |           |   |         |                        |       |          |
|---|-----------|---|---------|------------------------|-------|----------|
| $L_e, L_\mu, L_\tau, B, S, \tilde{B}, T, I_3 = 0$<br>$D_s^+ = c\bar{s}, D_s^- = \bar{c}s, D_s^{*+} = c\bar{s}, D_s^{*-} = \bar{c}s$ |           |   |         |                        |       |          |
|   | $M$ (MeV) | $\Gamma$ (MeV) ou $\tau$                  | $Q$     | $(S, C, \tilde{B}, T)$ | $I^G$ | $J^{PC}$ |
| $D_s^\pm$   | 1968.3(5) | $\tau = 490(9) \times 10^{-16} \text{ s}$ | $\pm 1$ | $(\pm 1, \pm 1, 0, 0)$ | 0     | $0^-$    |
| $D_s^{*\pm}$  | 2112.1(7) | $< 1.9$                                   | $\pm 1$ | $(\pm 1, \pm 1, 0, 0)$ | 0     |          |

| Mésons $c\bar{c}$   |              |                          |     |                        |       |          |
|---|--------------|--------------------------|-----|------------------------|-------|----------|
| $L_e, L_\mu, L_\tau, B, S, C, \tilde{B}, T, I_3, Q = 0$<br>$c\bar{c}$ |              |                          |     |                        |       |          |
|   | $M$ (MeV)    | $\Gamma$ (MeV) ou $\tau$ | $Q$ | $(S, C, \tilde{B}, T)$ | $I^G$ | $J^{PC}$ |
| $\eta_c(1S)$  | 2979.6(12)   | $17.3^{(+27)}_{(-26)}$   | 0   | $(0, 0, 0, 0)$         | $0^+$ | $0^{-+}$ |
| $J/\Psi(1S)$  | 3096.016(11) | 91.0(32)                 | 0   | $(0, 0, 0, 0)$         | $0^-$ | $1^{--}$ |

# 1.3 Hadrons

A more exhaustive list... with only a few examples

| Baryons ( $S = 0, I = \frac{1}{2}$ )                                       |              |                                |       |                        |               |                 |
|--|--------------|--------------------------------|-------|------------------------|---------------|-----------------|
| $L_e, L_\mu, L_\tau, S, C, \tilde{B}, T = 0, B = 1, I_3 = Q - \frac{1}{2}$ |              |                                |       |                        |               |                 |
| $p, N^+ = uud, \quad n, N^0 = udd$   |              |                                |       |                        |               |                 |
|  | $M$ (MeV)    | $\Gamma$ (MeV) ou $\tau$       | $Q$   | $(S, C, \tilde{B}, T)$ | $I^G$         | $J^{PC}$        |
| $p$  | 938.27203(8) | $\tau > 10^{31} - 10^{33}$ ans | +1    | (0, 0, 0, 0)           | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
| $n$  | 939.56536(8) | $\tau = 885.7$ (8) s           | 0     | (0, 0, 0, 0)           | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
| $N(1440) P_{11}$   | 1430 - 1470  | 250 - 450                      | +1, 0 | (0, 0, 0, 0)           | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
| $N(1520) D_{13}$   | 1515 - 1530  | 110 - 135                      | +1, 0 | (0, 0, 0, 0)           | $\frac{1}{2}$ | $\frac{3}{2}^-$ |

| Baryons $\Delta$ ( $S = 0, I = \frac{3}{2}$ )   |             |                          |               |                        |               |                 |
|---|-------------|--------------------------|---------------|------------------------|---------------|-----------------|
| $L_e, L_\mu, L_\tau, S, C, \tilde{B}, T = 0, B = 1, I_3 = Q - \frac{1}{2}$            |             |                          |               |                        |               |                 |
| $\Delta^{++} = uuu, \quad \Delta^+ = uud, \quad \Delta^0 = udd, \quad \Delta^- = ddd$ |             |                          |               |                        |               |                 |
|   | $M$ (MeV)   | $\Gamma$ (MeV) ou $\tau$ | $Q$           | $(S, C, \tilde{B}, T)$ | $I^G$         | $J^{PC}$        |
| $\Delta(1232) P_{33}$   | 1232 - 1234 | 115 - 125                | +2, +1, 0 - 1 | (0, 0, 0, 0)           | $\frac{3}{2}$ | $\frac{3}{2}^+$ |
| $\Delta(1600) P_{33}$   | 1550 - 1700 | 250 - 450                | +2, +1, 0 - 1 | (0, 0, 0, 0)           | $\frac{3}{2}$ | $\frac{3}{2}^+$ |

| Baryons $\Lambda$ ( $S = -1, I = 0$ )                    |              |   |     |                        |       |                 |
|--|--------------|---|-----|------------------------|-------|-----------------|
| $L_e, L_\mu, L_\tau, C, \tilde{B}, T, I_3, Q = 0, B = 1$ |              |   |     |                        |       |                 |
| $\Lambda^0 = uds$  |              |   |     |                        |       |                 |
|  | $M$ (MeV)    | $\Gamma$ (MeV) ou $\tau$                | $Q$ | $(S, C, \tilde{B}, T)$ | $I^G$ | $J^{PC}$        |
| $\Lambda$  | 1115.683 (6) | $\tau = 2.632$ (20) $\times 10^{-10}$ s | 0   | (-1, 0, 0, 0)          | 0     | $\frac{1}{2}^+$ |
| $\Lambda(1405) S_{01}$                                   | 1406 (4)     | 50.0 (20)                               | 0   | (-1, 0, 0, 0)          | 0     | $\frac{1}{2}^-$ |

## 2. Hadron spectroscopy

Introduction:

- 1<sup>st</sup> observation : invariance of the strong interactions w.r.t. the electric charge ( $p-p$ ,  $p-n$ ,  $n-n$  are equivalent for the strong interactions: underlying symmetry?)
- 2<sup>nd</sup> observation : masses identity

$$m_p = 938.3 \text{ (MeV)} \quad m_N = 939.1 \text{ (MeV)}$$

$$m_{\pi^\pm} = 139.6 \text{ (MeV)} \quad m_{\pi^0} = 135.0 \text{ (MeV)}$$

$$m_{K^\pm} = 493.7 \text{ (MeV)} \quad m_{K^0} = 497.7 \text{ (MeV)}$$

|| Those masses would have been probably degenerate in absence of e.m. interactions (symmetry violation analog to the Zeeman effect)

## 2.1 Isospin symmetry

Generalization to multiplets :

| Multiplet $J^P = 0^-$ |                     |           | Multiplet $J^P = 1^-$    |              |                 |
|-----------------------|---------------------|-----------|--------------------------|--------------|-----------------|
| Mésons                | Mass(MeV)           | Nom       | Mésons                   | Mass(MeV)    | Nom             |
| $\pi^+, \pi^0, \pi^-$ | 139.6, 135.0, 139.6 | pion      | $\rho^+, \rho^0, \rho^-$ | 768.5        | rho             |
| $K^+, K^0$            | 493.7, 497.7        | kaon      | $\omega$                 | 781.9        | oméga           |
| $\bar{K}^0, K^-$      | 497.7, 493.7        | antikaon  | $K^{*+}, K^{*0}$         | 891.6, 896.1 | kaon étoile     |
| $\eta$                | 547.5               | eta       | $\bar{K}^{*0}, K^{*-}$   | 896.1, 891.6 | antikaon étoile |
| $\eta'$               | 957.8               | eta prime | $\phi$                   | 1019.4       | phi             |

| Multiplet $J^P = \frac{1}{2}^+$ |                        |         | Multiplet $J^P = \frac{3}{2}^+$             |                        |              |
|---------------------------------|------------------------|---------|---|------------------------|--------------|
| Baryons                         | Mass(MeV)              | Nom     | Baryons                                     | Mass(MeV)              | Nom          |
| $p, n$                          | 938.3, 939.6           | nucléon | $\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$ | $\approx 1232$         | delta        |
| $\Lambda$                       | 1115.7                 | lambda  | $\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$     | 1382.8, 1383.7, 1387.2 | sigma étoile |
| $\Sigma^+, \Sigma^0, \Sigma^-$  | 1189.4, 1192.6, 1197.4 | sigma   | $\Xi^{*0}, \Xi^{*-}$                        | 1530.8, 1535.0         | xi étoile    |
| $\Xi^0, \Xi^-$                  | 1314.9, 1321.3         | xi      | $\Omega^-$                                  | 1672.5                 | oméga        |

## 2.1 Isospin symmetry

Conclusion: hadrons can be classified as multiplets of ~equal masses particles differing by their electrical charge :

$$\begin{array}{l}
 \text{singlets :} \quad \eta, \eta', \omega, \phi, \Lambda, \Omega^- \\
 \text{doublets :} \quad \begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}, \begin{pmatrix} K^{*+} \\ K^{*0} \end{pmatrix}, \begin{pmatrix} \bar{K}^{*0} \\ K^{*-} \end{pmatrix}, \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \\
 \text{triplets :} \quad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}, \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}, \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} \\ \Sigma^{*0} \\ \Sigma^{*-} \end{pmatrix} \\
 \text{quadruplets :} \quad \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}
 \end{array}$$

From the point of view of strong interactions, proton and neutron are almost the same particle. One creates an abstract space in which strong interactions are invariant under rotations...



## 2.1 Isospin symmetry

The conserved quantity (Noether's theorem) is called **isospin**.

Isospin treatment follows the kinetic moment one's and relies on the **group theory** (SU(2) representation) :

- In the isospin space one introduces an operator  $\vec{I} = (I_1, I_2, I_3)$  which commutation rules read  $[I_i, I_j] = \epsilon_{ijk} I_k$
- The eigenstates  $|I, I_3\rangle$  of the observables  $I^2, I_3$  are such that :

$$I^2 |I, I_3\rangle = I(I+1) |I, I_3\rangle$$

$$I_3 |I, I_3\rangle = I_3 |I, I_3\rangle$$

- A multiplet has  $2I+1$  eigenstates  $I_3 = -I, \dots, +I$  (“ $2I+1$ ”-plet)

## 2.1 Isospin symmetry

Within such representations one has the following multiplets:

$$|p\rangle = \left| I = \frac{1}{2}, I_3 = \frac{1}{2} \right\rangle = \left| \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$|n\rangle = \left| \frac{1}{2}, -\frac{1}{2} \right\rangle,$$

*Doublet*

$$|\Delta^{++}\rangle = \left| \frac{3}{2}, \frac{3}{2} \right\rangle$$

$$|\Delta^+\rangle = \left| \frac{3}{2}, \frac{1}{2} \right\rangle$$

$$|\Delta^0\rangle = \left| \frac{3}{2}, -\frac{1}{2} \right\rangle$$

$$|\Delta^-\rangle = \left| \frac{3}{2}, -\frac{3}{2} \right\rangle$$

*Quadruplet*

## 2.1 Isospin symmetry

Let's define an operator acting on the isospin multiplet

$$|\Phi'\rangle = U|\Phi\rangle$$

- $U$  must be a unitary complex matrix  $U^\dagger U = 1$
- unimodular  $\det(U) = 1$
- of rank 1 (only 1 matrix simultaneously diagonalizable)
- ➔ Those matrices are known : the  $2 \times 2$  special unitary matrices from SU(2) group (**spin** representation)

## 2.2 Basics of Lie groups

SU(N) groups :

- **Group generators** :  $I_k$  defined such as  $U(\alpha_1, \alpha_2, \alpha_3) = e^{-i\alpha_k I_k}$   
# of independent generators :  $m = n^2 - 1$
- **Properties** :  $I_k^\dagger = I_k$  and  $Tr(I_k) = 0$
- # of generators simultaneously diagonalizable (**rank**) :  $r = n - 1$
- # of **Casimir operators** (function of the generators commuting with all of them) :  $r = n - 1$   
ex. SU(2) :  $I^2 = I_1^2 + I_2^2 + I_3^2$
- The **fundamental representation** is of dimension N :  $\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{pmatrix}$
- **Structure constants** defined by:  $[I_i, I_j] = f_{ijk} I_k$

## 2.2 Basics of Lie groups

SU(2) (spin-isospin...)

- Generators proportional to the Pauli matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Fundamental representation (doublet) : each state can be build on basis vectors

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = X_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + X_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Structure constants  $f_{ijk} = \epsilon_{ijk}$  (permutation operator)

## 2.2 Basics of Lie groups

Product of fundamental representations:

- Like the spin couplings

$$|I_a, I_b, I, I_3\rangle = \sum_{I_{a3}, I_{b3}} C(I_a, I_b, I, I_3) |I_a, I_{a3}, I_b, I_{b3}\rangle$$

- Example in SU(2):  $\frac{\vec{1}}{2} + \frac{\vec{1}}{2} = \vec{0} + \vec{1}$  or  $2 \otimes 2 = 1 \oplus 3$
- Generation of an isospin triplet : example of pions:

$$|\pi^+\rangle \equiv |1, 1\rangle = \left| \frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$|\pi^0\rangle \equiv |1, 0\rangle = \frac{1}{\sqrt{2}} \left( \left| \frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{-1}{2} \right\rangle + \left| \frac{1}{2}, \frac{1}{2}; \frac{-1}{2}, \frac{1}{2} \right\rangle \right)$$

$$|\pi^-\rangle \equiv |1, -1\rangle = \left| \frac{1}{2}, \frac{1}{2}; \frac{-1}{2}, \frac{-1}{2} \right\rangle$$

- + a remaining isospin singlet

## 2.2 Basics of Lie groups

Gell-Mann Nishijima relation :

- Strong interactions are invariant under a rotation around the 3<sup>rd</sup> component of the isospin
- $I_3$  value is also a measurement of the charge eigenstate inside the multiplet

➔ There should be a link between  $I_3$  and  $Q$

$$Q = I_3 + \frac{Y}{2} \quad (Y = B + s + c + b + t)$$

The diagram illustrates the Gell-Mann Nishijima relation  $Q = I_3 + \frac{Y}{2}$  and the definition of hypercharge  $Y = B + s + c + b + t$ . Three boxes with arrows point to the terms in the equation:

- A box labeled **Invariant through all interactions** points to  $Q$ .
- A box labeled **Invariant through strong Interactions only** points to  $I_3$ .
- A box labeled **Hypercharge** points to  $Y$ .

## 2.2 Basics of Lie groups

Adjoint representation:

- In SU(2),  $p$  and  $n$  form an isospin doublet, what about  $\bar{p}$  et  $\bar{n}$ ?  
In isospin space let's take a rotation around the 2<sup>nd</sup> axis :

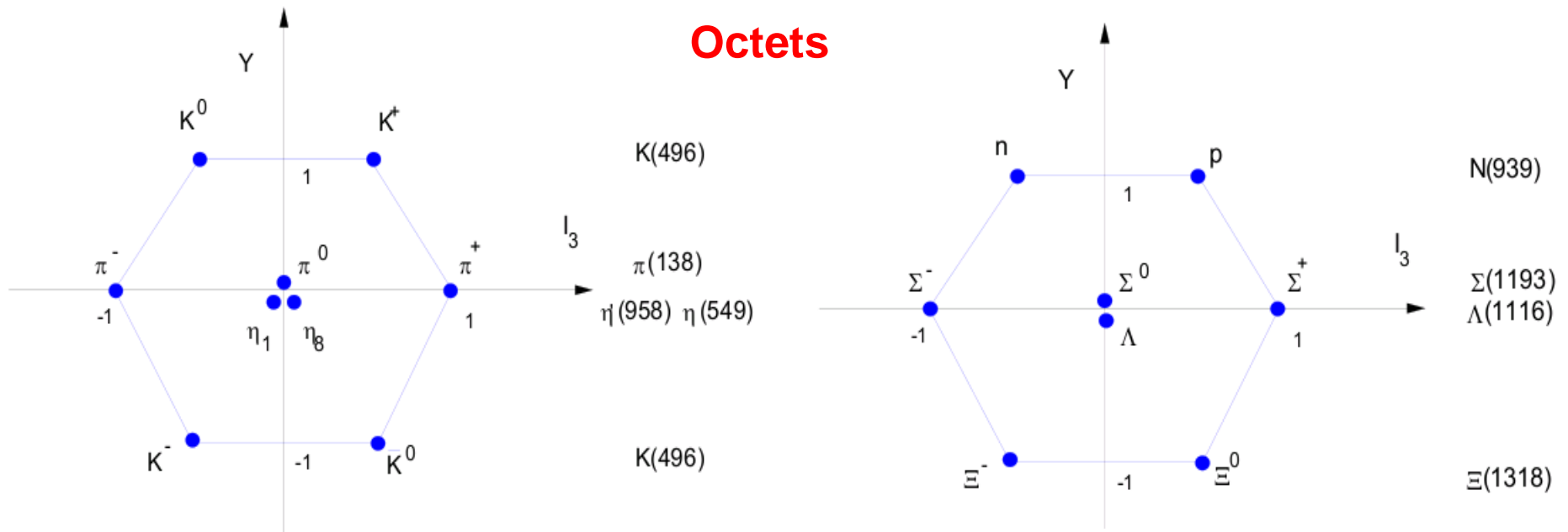
$$U(\theta) = 1 - i\theta \frac{\tau_2}{2} + \frac{1}{2!} \left( i\theta \frac{\tau_2}{2} \right)^2 + \dots = \cos \frac{\theta}{2} \cdot 1 - i \sin \frac{\theta}{2} \cdot \tau_2$$

- Rotated states : 
$$\begin{pmatrix} p' \\ n' \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$$
- Gell-Mann Nishijima relation:  $Q(\bar{p}) = -Q(p) = -1 \quad Q(\bar{n}) = 0$   
 $B(\bar{p}) = B(p) = -1$   
 $\Rightarrow I_3(\bar{p}) = -\frac{1}{2} \quad I_3(\bar{n}) = +\frac{1}{2}$
- Adjoint representation: 
$$\begin{pmatrix} \bar{n}' \\ -\bar{p}' \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \bar{n} \\ -\bar{p} \end{pmatrix}$$



## 2.3 Quarks model

- The adjoint representation is generated by the following matrices:  $(I_i)_{jk} = i f_{ijk}$
- Quark model: each hadron multiplet is characterized by a baryonic number, a spin and a defined parity or equivalently by an hypercharge and the 3<sup>rd</sup> component of the isospin :



## 2.3 Quarks model

- In a constant-Y row the masses are nearly equal. Gell-Mann and Zweig proposed in 1964 that hadrons were not elementary but compounds of 3 types of quarks : u, d, s.
- In terms of group theory this model will be read through **SU(3)** representations. To be complete the model has to be extended to 6 flavors (including c, b, t) and convoluted with a space of colors (3 colors: R G B, already mentioned)
- The SU(3) group. **Triplet** fundamental representation :  $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ .

Basis vectors :

|       |   |   |   |
|-------|---|---|---|
|       | $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ | $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ | $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ |
| $I_3$ | $\frac{1}{2}$                               | $-\frac{1}{2}$                              | $0$   |
| $Y$   | $\frac{1}{3}$                               | $\frac{1}{3}$                               | $-\frac{2}{3}$                              |

## 2.3 Quarks model

- Gell-Mann matrices (8 generators)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- Structure constants

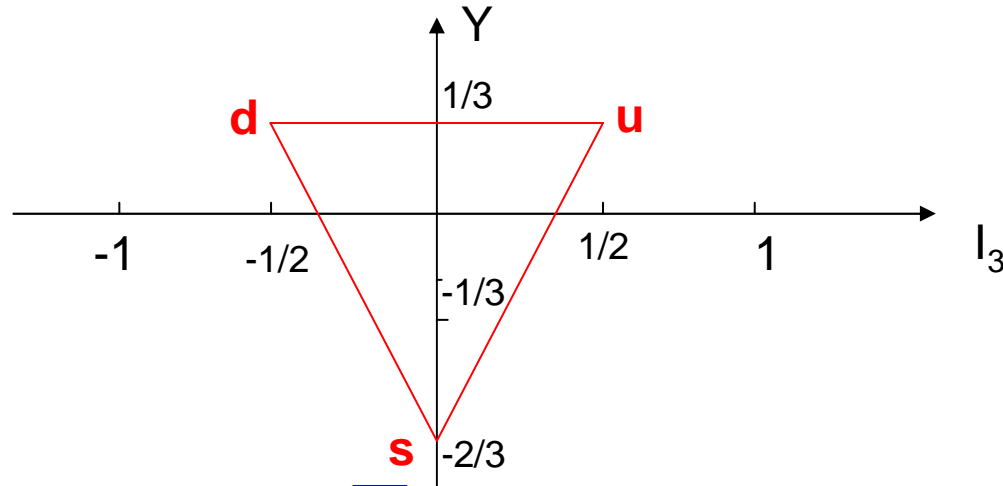
$$f_{123} = 1$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

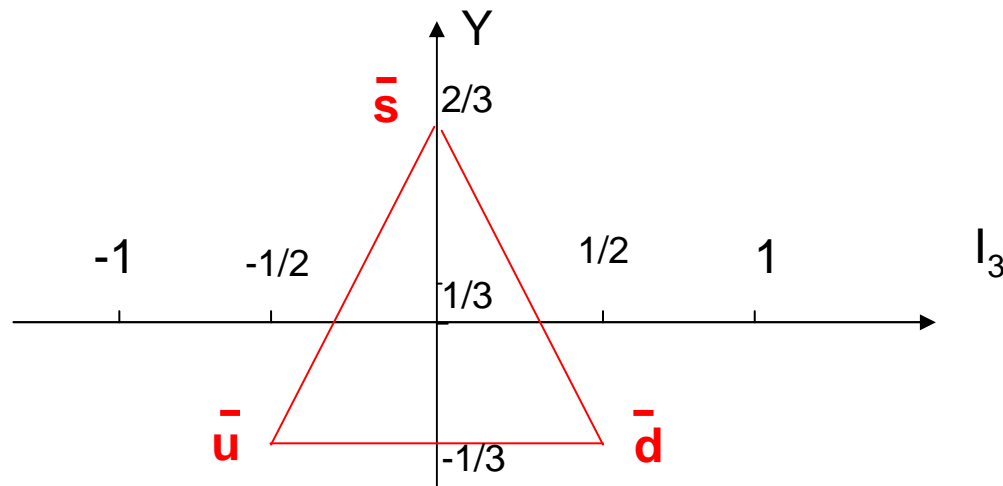
$$f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}$$

## 2.3 Quarks model

- Fundamental representation  $3$  :



- Adjoint representation  $\bar{3}$  :



## 2.3 Quarks model

- How do we generate the hadron spectrum?
- For mesons :  $3 \otimes \bar{3} = 1 \oplus 8$  (singlet + octet)
- For baryons :  $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8' \oplus 10$  (singlet + octets + decuplet)
- The construction of the wavefunctions is rather technical (proper antisymmetrization etc) but the principle is there...

## 2.3 Quarks model

- Examples in the mesonic sector
- Quark contents:

Octet :

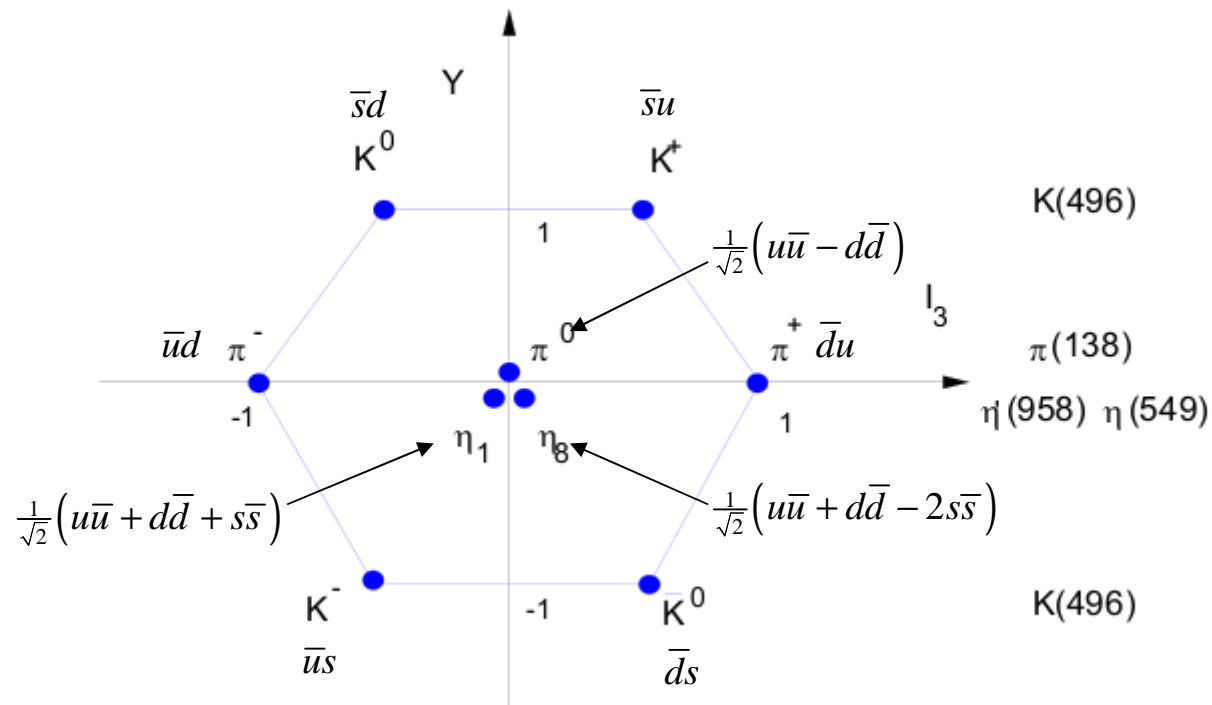
| quark-antiquark dans $SU(3)$ , états mixtes |   |
|---|---|
| Constituants                                | $\mathbf{8}_M$  |
| $\pi^+(u\bar{d})$                           | $u\bar{d}$  |
| $\pi^0(u\bar{u}, d\bar{d})$                 | $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$             |
| $\pi^-(d\bar{u})$                           | $d\bar{u}$  |
| $K^+(u\bar{s})$                             | $u\bar{s}$  |
| $K^0(d\bar{s})$                             | $d\bar{s}$  |
| $\bar{K}^0(s\bar{d})$                       | $s\bar{d}$  |
| $K^-(s\bar{u})$                             | $s\bar{u}$  |
| $\eta_8(u\bar{u}, d\bar{d}, s\bar{s})$      | $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ |

Singlet :

| quark-antiquark dans $SU(3)$ , états antisymétriques |  |
|--|--|
| Constituants   | $\mathbf{1}_A$                                       |
| $\eta_1(u\bar{u}, d\bar{d}, s\bar{s})$               | $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ |

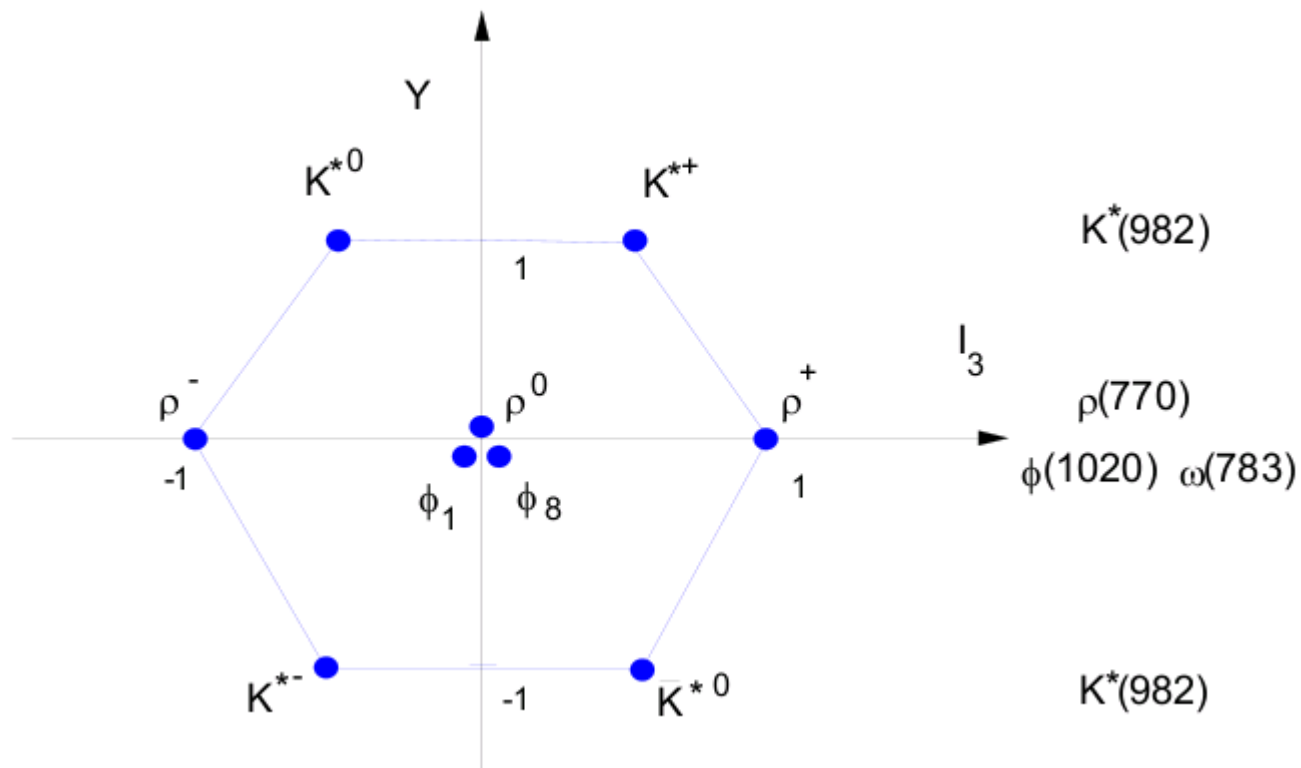
## 2.3 Quarks model

- Pseudo-scalar mesons diagram (octet + singlet):  $J^P = 0^-$



## 2.3 Quarks model

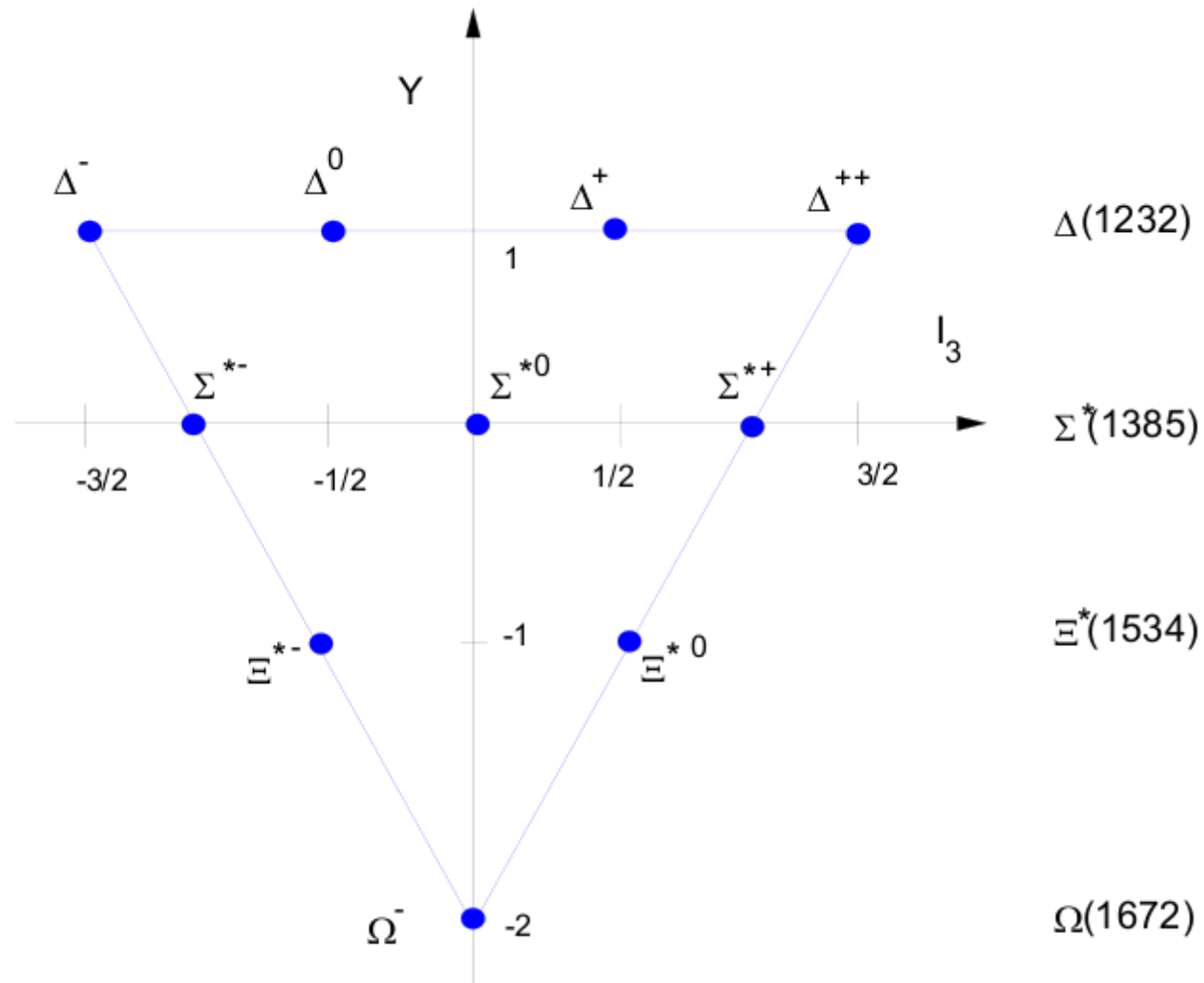
- Pseudo-vector mesons diagram (octet + singlet): same quarks content but // spin :  $J^P = 1^-$





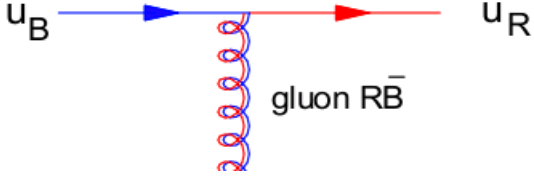
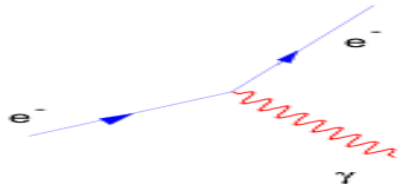
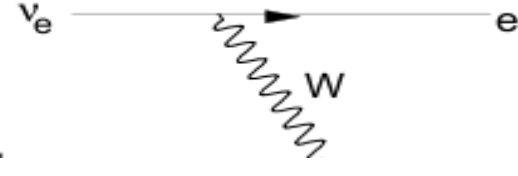
## 2.3 Quarks model

- Just for fun : baryons decuplet



# 3- Fundamental interactions

- Leptons and quarks interactions are mediated by specific gauge bosons :

|                    |          |                  |  |  |
|--------------------|----------|------------------|--|--|
| <b>strong</b>      | gluons   | $g (8)$          | $M=0$                                    |   |
| <b>e.m.</b>        | photon   | $\gamma (1)$     | $M=0$                                    |   |
| <b>weak</b>        |          | $Z^0$<br>$W^\pm$ | $M=90 \text{ GeV}$<br>$M=81 \text{ GeV}$ |  |
| <b>gravitation</b> | graviton | $h^{\mu\nu}$     | ?  | ?  |

## 3.1- Range and propagators

- Yukawa approach: in 1935 it was proposed a link between the range of an interaction and the mass of the “carrier” quantum.
- Heisenberg inequality :

$$R = c\Delta t \approx \frac{1}{\Delta E} = \frac{1}{m}$$

- Formally (Klein-Gordon equation ie “massive” photon propagation equation):  $\square\psi + m^2\psi = 0$

For a static spherical potential :

$$\Delta\tilde{\psi}(r) + m^2\tilde{\psi}(r) = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} (\tilde{\psi}(r)) \right) + m^2\tilde{\psi}(r) = 0$$

$$\Rightarrow \tilde{\psi}(r) = \frac{g}{4\pi r} e^{-r/R} \quad \text{where} \quad R = \frac{1}{m}$$

## 3.1- Range and propagators

- Historically this approach led to the prediction of an intermediate quantum for the strong interaction of mass close to:

$$m = \frac{1}{R} \sim \frac{1}{\text{few fm}} \sim 100 - 200 \text{ MeV}$$

- The **pion** was then discovered (140 MeV) which can be seen as the carrier for the residual strong interaction between nucleons (not quarks) at the scale of the nucleus.
- Its small mass is the manifestation of another symmetry breaking, the **chiral symmetry**  $SU(2)_L \times SU(2)_R$ .
- This simple model gives an idea of the **range** of an interaction and the link with the intermediate properties.

## 3.1- Range and propagators

- The Fourier transform of the Yukawa potential is given by :

$$\tilde{\psi}(r) \propto \frac{e^{-r/R}}{r} \quad \text{where} \quad R = \frac{1}{m}$$

$$\Rightarrow \tilde{\psi}(q) \propto \frac{1}{\vec{q}^2 + m^2 - i\epsilon}$$

- The quantity obtained is called the **propagator**.
- Using the **Green function formalism** for the Klein-Gordon equation:  $(\square + m^2)\psi(x) = 0$  where  $x = x^\mu = (x^0, \vec{x})$

$$\Rightarrow (q^2 - m^2)\psi(q) = 0 \quad \text{with} \quad q = q^\mu = (q^0, \vec{q})$$

$$\Rightarrow (q^2 - m^2)G(q) = \delta^4(q)$$

$$\Rightarrow G(q) = \frac{\delta^4(q)}{q^2 - m^2} \Rightarrow \text{Propagator} = \frac{i}{q^2 - m^2}$$

## 3.1- Range and propagators

- What is the role of the integration constant  $g$ ?

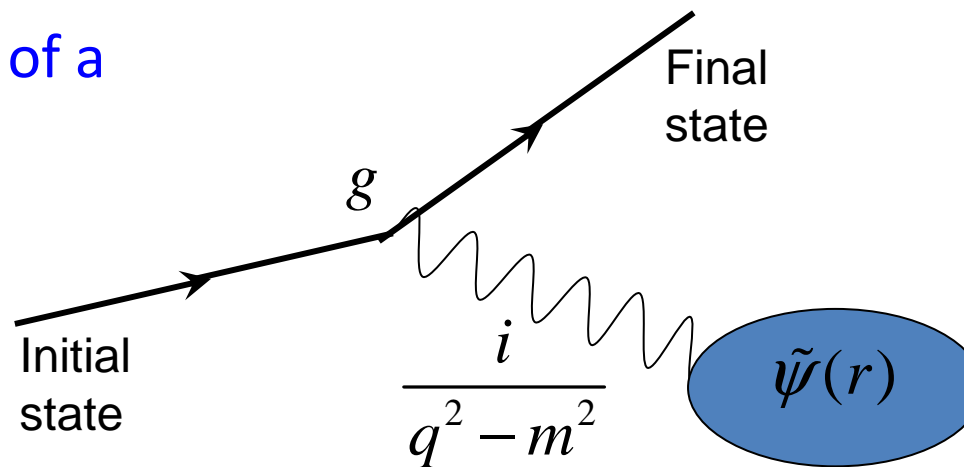
$$\tilde{\psi}(r) = \frac{g}{4\pi r} e^{-r/R}$$

- In Electromagnetism we have the standard Coulomb potential :

$$\tilde{\psi}(r) = \frac{Q}{4\pi r}$$

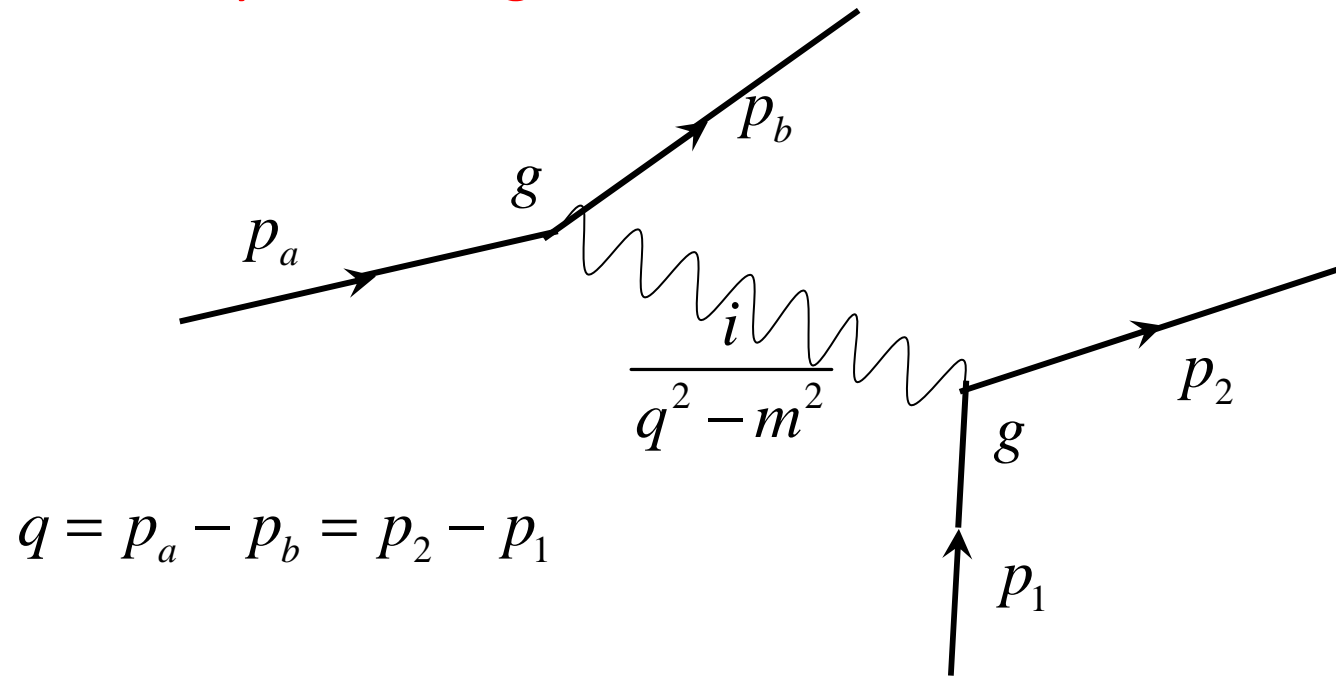
corresponding to a vanishing mass (for photon) or infinite range

- $Q$  and  $g$  play the role of a **coupling constant** for the interaction



## 3.1- Range and propagators

- Basic Feynman diagram :



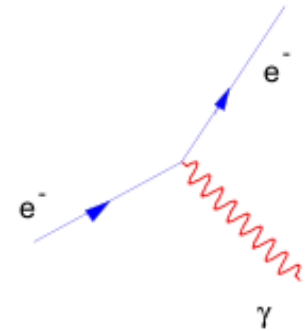
- The amplitude of the process writes:

$$g^2 A(p_a, p_b) \frac{1}{q^2 - m^2} A(p_1, p_2) \delta^4(p_a + p_1 - p_b - p_2)$$

## 3.2- Electroweak interactions

### ELECTROMAGNETIC INTERACTIONS

- Coupling constant :  $\alpha_{em} = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$
- Example : Rutherford cross-section  $\frac{\partial\sigma}{\partial q^2} \propto \frac{\alpha_{em}^2}{q^4}$
- Typical cross-section  $\sim 10^{-33} \text{ m}^2$
- Typical interaction times  $\sim 10^{-20} \text{ s}$
- Photon ( $\gamma$ ) exchange, infinite range

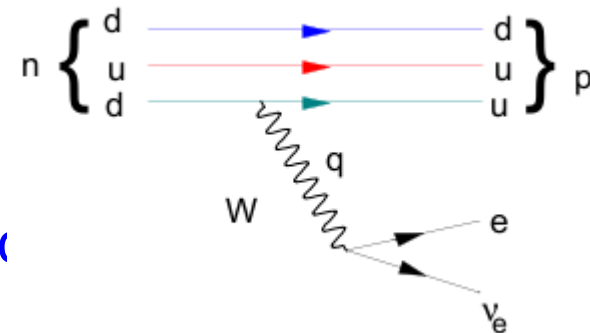




## 3.2- Electroweak interactions

### WEAK INTERACTIONS

- Coupling constant :  $\alpha_{Fermi} = \frac{G_F m_P^2}{4\pi} \approx 10^{-6}$
- Example : neutron  $\beta$ -decay
- Weak interactions do not conserve the (
- Typical cross-section  $\sim 10^{-44} \text{ m}^2$
- Typical interaction times  $\sim 10^{-10} \text{ s}$
- Weak bosons exchange, finite range



## 3.2- Electroweak interactions

### WEAK INTERACTIONS (cont'd)

- Typical range :  $M \sim 80 - 90 \text{ GeV} \Rightarrow R \sim 10^{-18} \text{ m}$
- Due to the large mass of the exchanged bosons the weak interactions can often be considered point-like

$$g_W^2 \frac{1}{q^2 - M_W^2} \xrightarrow{q^2 \rightarrow 0} \frac{g_W^2}{M_W^2} \equiv G_F = 10^{-5} \text{ GeV}^{-2}$$

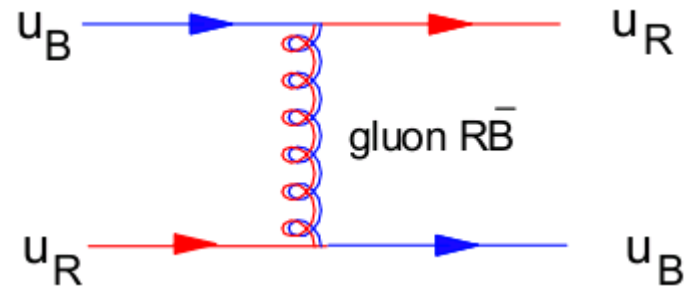
- Weinberg angle (electroweak theory) :

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{e^2}{4\pi \sin^2 \theta_W} = \frac{1}{29} \quad (\sin^2 \theta_W = 0.22)$$

## 3.3- Strong interactions

### Features

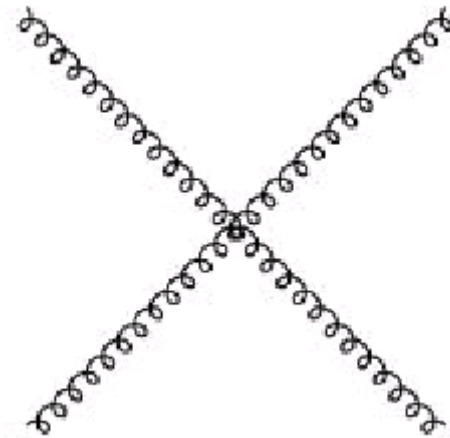
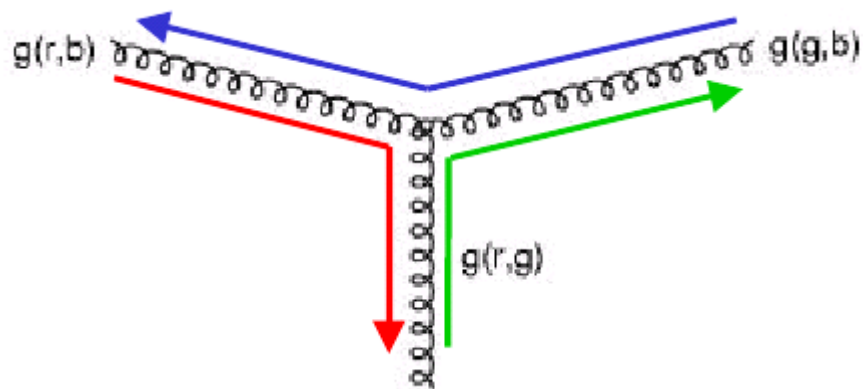
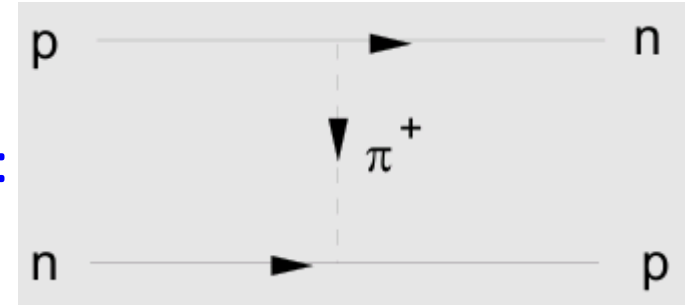
- Coupling constant :  $\alpha_s \simeq 1$
- Carry a colored charge
- Typical cross-section  $\sim 10^{-30} \text{ m}^2$
- Typical interaction times  $\sim 10^{-23} \text{ s}$
- Gluons (g) exchange, effective finite range due to the confinement  $R \sim 10^{-15} \text{ m}$  . Asymptotic freedom.



## 3.3- Strong interactions

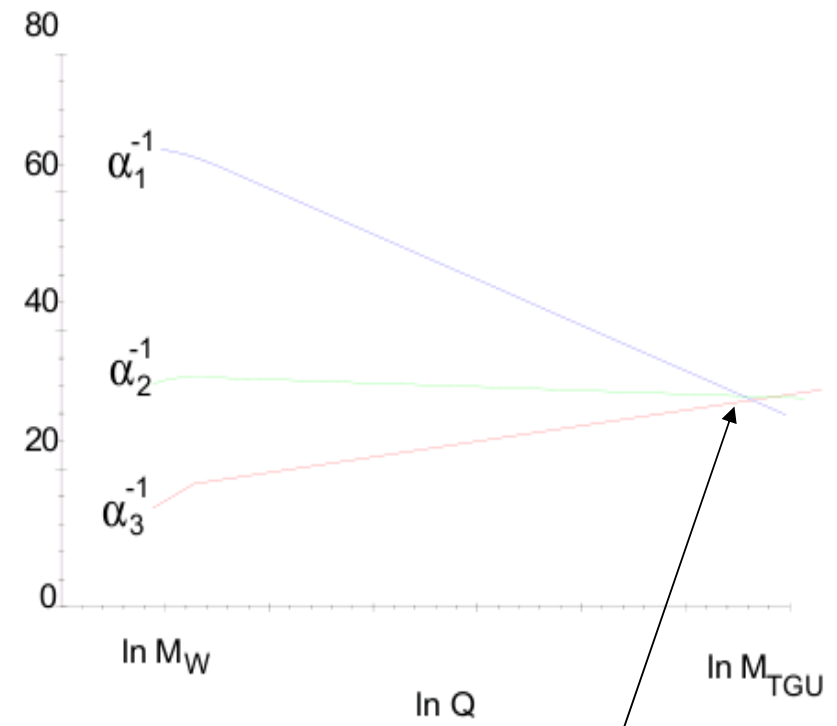
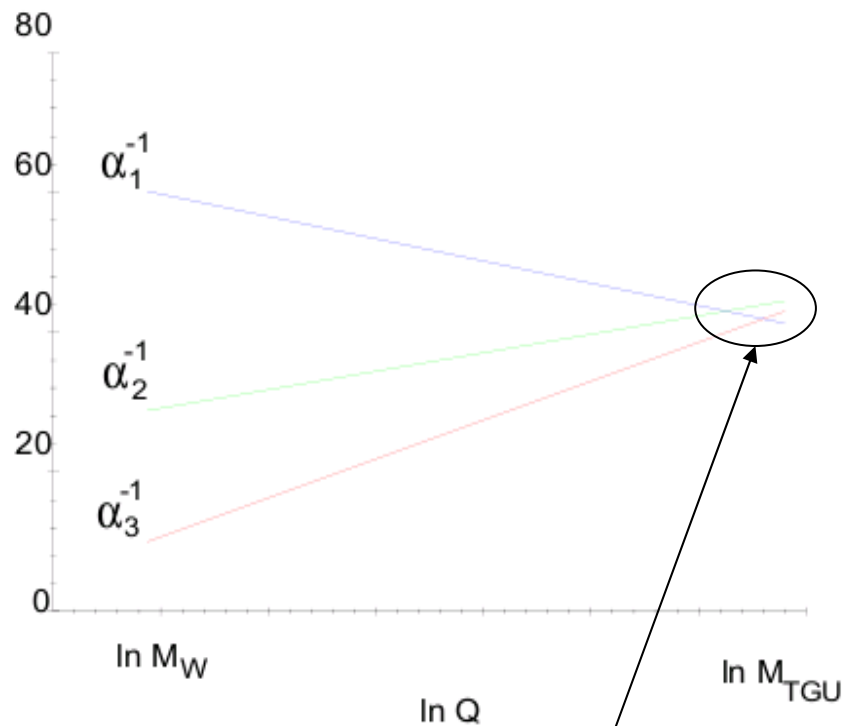
Features (cont'd)

- Residual interaction (at the nuclei scale) :
- $\exists$  couplings between gluons :



## 3.3- Towards unification?

- Coupling constants vary with the energy



- At large scales all couplings become nearly equal.
- In some “beyond Standard Model” models (SUSY etc) this occurs!