Chapter 1
Particles...

How do we classify them?
How do they interact?
How do we detect them?

Outline/Plan

1. Fundamental particles
   1. Leptons
   2. Quarks
   3. Hadrons

2. Hadron spectroscopy
   1. Isospin symmetry
   2. Basics of group theory. The SU(N) group.
   3. The quark model

3. Fundamental interactions
   1. Range and propagators
   2. Electro-weak interaction
   3. Strong interaction

1. Particules fondamentales
   1. Leptons
   2. Quarks
   3. Hadrons

2. Spectroscopie hadronique
   1. La symétrie d'isospin
   2. Rappel de théorie des groupes. Le groupe SU(N).
   3. Le modèle des quarks.

3. Interactions fondamentales
   1. Portée d'une interaction et propagateurs.
   2. Interaction électro-faible.
   3. Interaction forte.

1- Fundamental particles

General features:

- Fundamental particles can not be separated into smaller components (elementary particles such as: electron, photon, quarks…)
- Some particles are composite ones (protons and neutrons are composed of 3 quarks, pions of 1 quark and 1 anti-quark…)
- There are 2 ways of classifying the particles:

1. Following the spin-statistics

Fermions (1/2 integer spin, Fermi-Dirac statistics)

Vs

Bosons (integer spin, Bose-Einstein statistics)

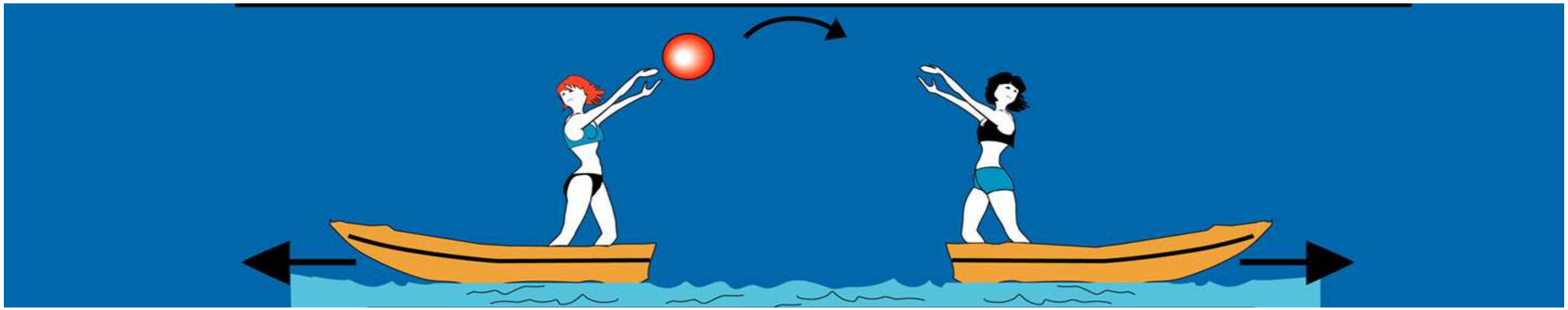
2. Following the interaction(s) they are sensitive to…

1- Fundamental particles

There are 4 fundamental interactions:

- Strong interaction (e.g. nuclei structure)
- Electromagnetic interaction (e.g. atomic physics, light, μ-wave…)
- Weak interaction (e.g. β radioactivity phenomena)
- Gravity (neglected at energy scales well below 10^{19}GeV)

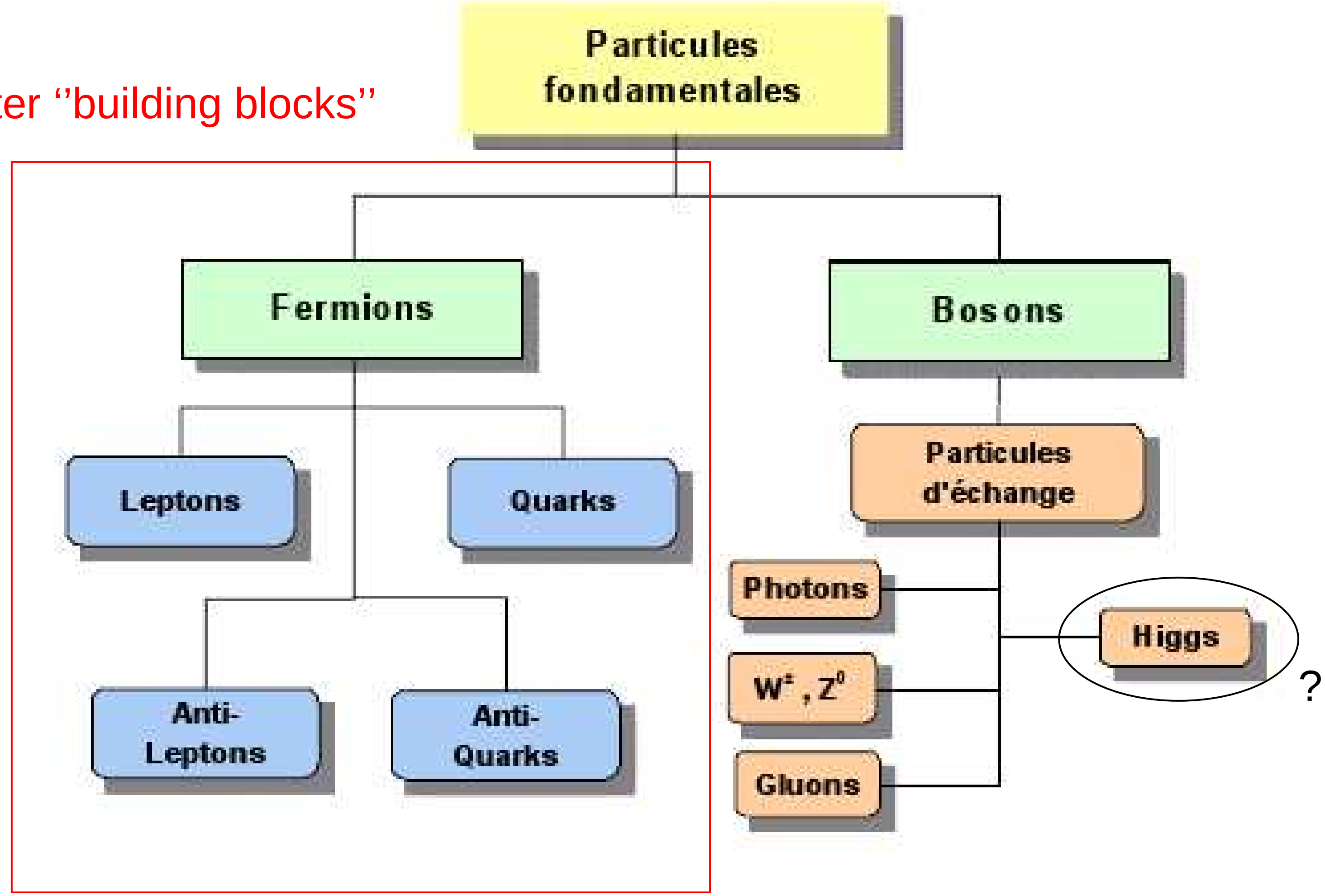
In quantum field theory any interaction is modeled by an intermediate particle exchange (gauge bosons).



1- Fundamental particles

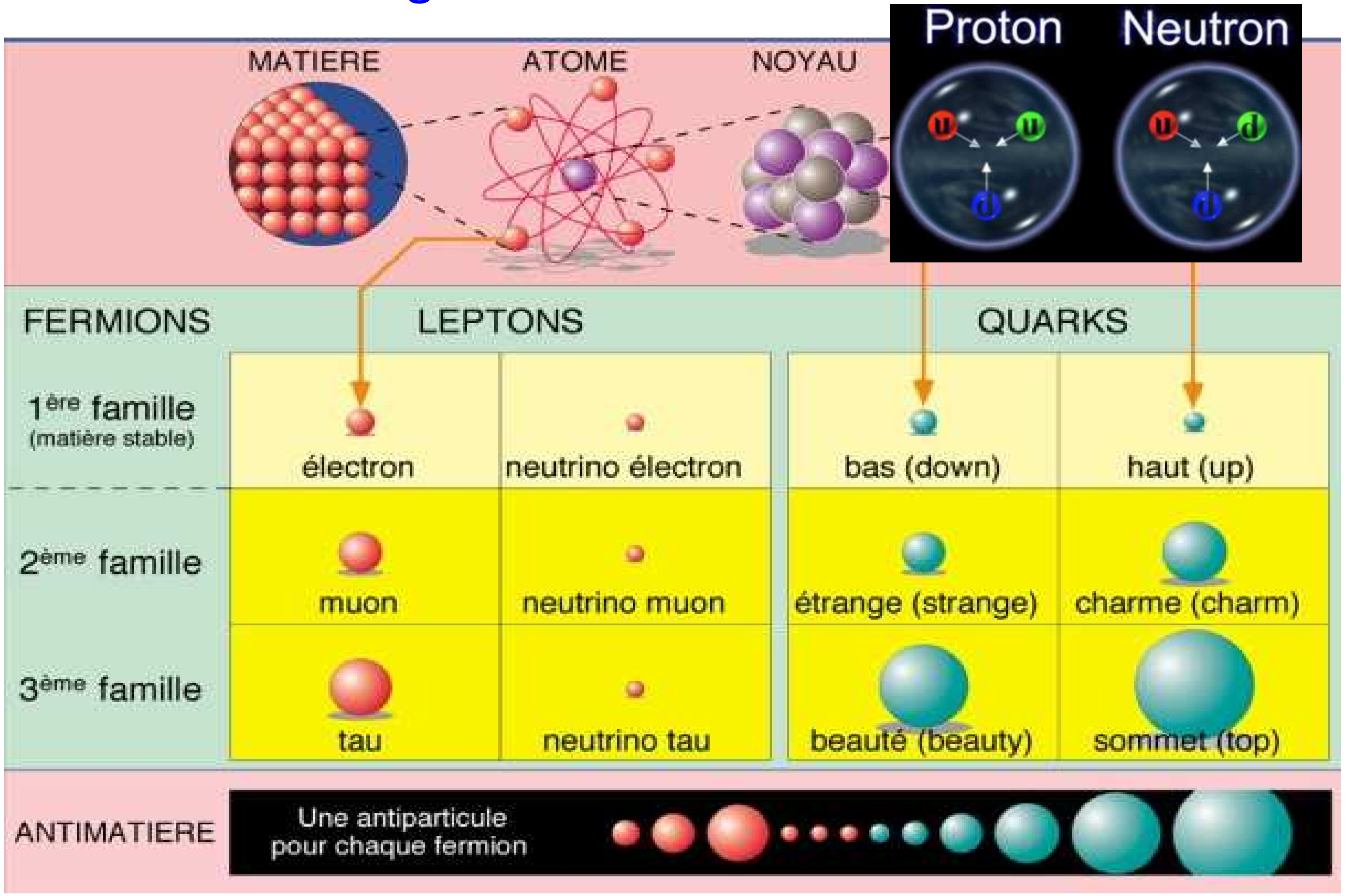
A simplified scheme

Matter ''building blocks''



1.1 Leptons and quarks

They are (up-to-now) the most elementary particles known and constitute the building blocks of atoms



1.1 Leptons

Leptons:

- Are insensitive to strong interaction
- Carry integer electric charges ($n \times 1.610^{-19}$C with $n \in \aleph$)
- Carry a "weak" charge ie can be associated in weak interaction doublets
- Are organized into 3 families : electron, muon, tau

Leptons (spin $\frac{1}{2}$)			
$Q = 0$ $Q = -1$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

- Muons and taus are "heavy" and unstable copies of electrons

1.1 Leptons

Leptonic number: global symmetry associated to leptons implying that 3 numbers are conserved additively in the interactions:

- Le = +1 (e^- and ν_e) / Le = -1 (e^+ and $\bar{\nu}_e$) / Le = 0 for others
- Lμ = +1 (μ^- and ν_μ) / Lμ = -1 (μ^+ and $\bar{\nu}_\mu$) / Lμ = 0 for others
- Lτ = +1 (τ^- and ν_τ) / Lτ = -1 (τ^+ and $\bar{\nu}_\tau$) / Lτ = 0 for others

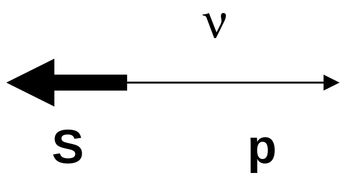
- Reactions example:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (L_\mu : 0 = -1 + (+1))$$
$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (L_e : 0 = +1 + (-1) / L_\mu : +1 = +1)$$

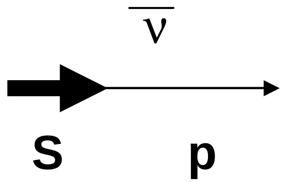
1.1 Leptons

Neutrinos are only sensitive to weak interactions and have a fixed

helicity (operator : $\lambda = \frac{\vec{S}.\vec{p}}{p}$)



Left-handed neutrinos



Right-handed anti-neutrinos

1.1 Leptons

Leptons summary

Leptons $S, C, \widetilde{B}, T, I, I_3 = 0$						
	M (MeV)	τ	Q	(L_e, L_μ, L_τ)	$(I^W, I_3^W)_{R,L}$	J^{PC}
e	$0.51099892(4)$	$> 4.6 \times 10^{26}$ ans	-1	$(1,0,0)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	$\frac{1}{2}$
ν_e	$< 3 \times 10^{-6}$	$> 300\ m_\nu$ s/eV	0	$(1,0,0)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	$\frac{1}{2}$
μ	$105.658369(9)$	$2.197030(4) \times 10^{-6}$ s	-1	$(0,1,0)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	$\frac{1}{2}$
ν_μ	< 0.19	$> 15.4\ m_\nu$ s/eV	0	$(0,1,0)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	$\frac{1}{2}$
τ	$1776.99^{(+29)}_{(-26)}$	$290.6(11) \times 10^{-15}$ s	-1	$(0,0,1)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	$\frac{1}{2}$
ν_τ	< 18.2	$-$	0	$(0,0,1)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	$\frac{1}{2}$

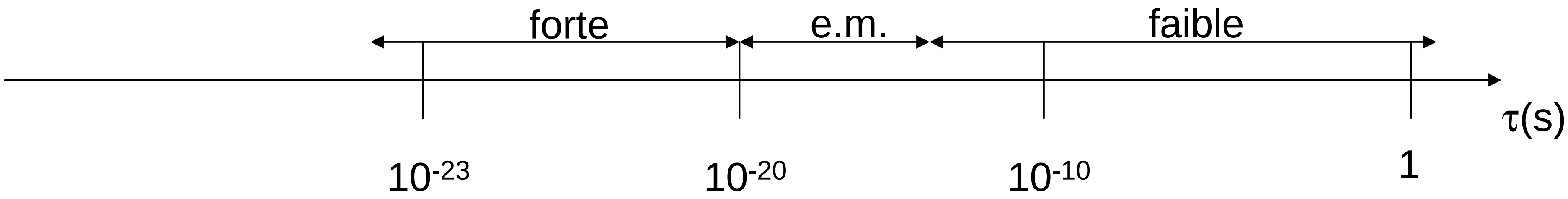
Notations:

- I^W and I_3^W are related to the weak isospin
- J^{PC} = Spin$^{\text{Parity C-Parity}}$

1.1 Leptons

What about stability and lifetime? Almost all particles (but e.g. electrons, protons) are unstable and decay with a time which depends on the type of interaction and the available phase space

Hierarchy:



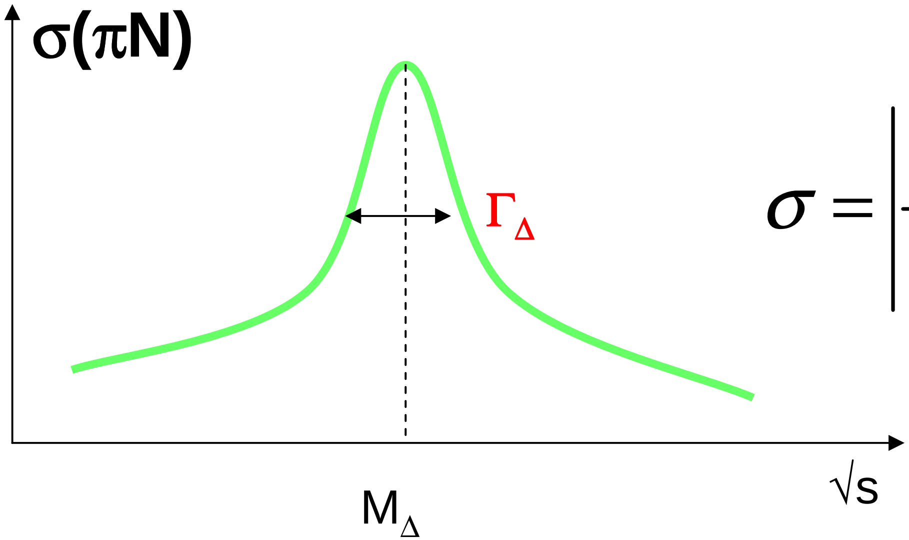
Width (energy scale):

$$\Delta M = \Gamma = \frac{1}{\tau} \quad \text{(MeV)}$$

1.1 Leptons

Strong decays and resonances :

$$\Delta \to \pi N \quad (\Gamma = 115\,\mathrm{MeV})$$
$$\rho \to \pi\pi \quad (\Gamma = 150\,\mathrm{MeV})$$



$$\sigma = \left| \frac{1}{(s^2 - M_\Delta s) + i M_\Delta \Gamma(s)} \right|^2$$

$$\tau \sim 1/100(MeV) * \left[200(MeV.fm)/3.10^8.10^{15}(fm.s^{-1}) \right]$$

$$\tau \sim 10^{-23} s$$

1.2 Quarks

Quarks:

- Are sensitive to strong interaction (they are the fundamental components of nuclear matter)
- Carry fractional electric charges (e.g. Qu=2/3×e)
- Carry a "weak" charge ie can be associated in weak interaction doublets
- Carry also a "colored charge" and are associated in triplets of the strong interaction
- Are organized into 3 families (as the leptons are, probable link?) which are ~identical but for the masses

Quarks			
$Q = \frac{2}{3}$	u(up)	c(charme)	t(top)
$Q = -\frac{1}{3}$	d(down)	s(étrange)	b(bottom)

1.2 Quarks

Quarks are confined : they can not be observed in a free state (extreme case: quark-gluon plasma)

Global quantum numbers are associated with the quark content of a compound : strangeness (s), charm (c), beauty (b), top (t)…

These numbers are conserved in all but the weak interactions

Quarks

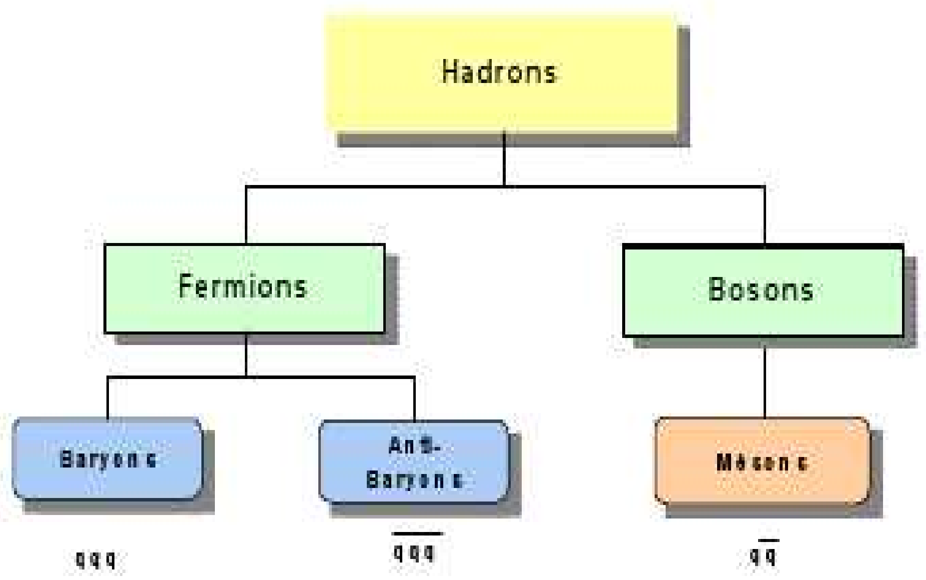
$$L_e, L_\mu, L_\tau = 0, I_3 = Q - \frac{1}{2}\left(B + S + C + \tilde{B} + T\right)$$

	M (MeV)	Q	B	$(S, C, \tilde{B}, T)$	$(I^W, I_3^W)_{R,L}$	I^g	J^{PC}
u	$1.5 - 4$	$\frac{2}{3}$	$\frac{1}{3}$	$(0,0,0,0)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	$\frac{1}{2}$	$\frac{1}{2}^+$
d	$4 - 8$	$-\frac{1}{3}$	$\frac{1}{3}$	$(0,0,0,0)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	$\frac{1}{2}$	$\frac{1}{2}^+$
s	$80 - 130$	$-\frac{1}{3}$	$\frac{1}{3}$	$(-1,0,0,0)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	0	$\frac{1}{2}^+$
c	$1150 - 1350$	$\frac{2}{3}$	$\frac{1}{3}$	$(0,1,0,0)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	0	$\frac{1}{2}^+$
b	$4100 - 4400$ $(\overline{MS})$ $4600 - 4900$ $(1S)$	$-\frac{1}{3}$	$\frac{1}{3}$	$(0,0,-1,0)$	$(0,0)_R, (\frac{1}{2}, -\frac{1}{2})_L$	0	$\frac{1}{2}^+$
t	$174.3(51)$	$\frac{2}{3}$	$\frac{1}{3}$	$(0,0,0,1)$	$(0,0)_R, (\frac{1}{2}, \frac{1}{2})_L$	0	$\frac{1}{2}^+$

1.3 Hadrons

Hadrons are the compound particles sensitive to the strong interaction. They are divided into 2 categories:

- Baryons : made of 3 quarks ($q_1q_2q_3$)
- Mesons : made of 1 quark and 1 anti-quark ($q_1\overline{q_2}$)



1.3 Hadrons

Examples:

Hadrons		
p	proton	uud
n	neutron	udd
π^+, π^0, π^+	pions	$u\bar{d}, \; u\bar{u}+d\bar{d}, \; \bar{u}d$
ρ^+, ρ^0, ρ^-	mésons ρ	
Λ	lambda	udc
$K^+, K^0, \bar{K}^0, K^-$	mésons K	$u\bar{s}, \; d\bar{s}, \; s\bar{d}, \; \bar{u}s$

1.3 Hadrons

- Hadrons carry integer electric charge
- They interact weakly
- We associate a global quantum number (baryonic number), conserved additively in all reactions and defined as :

  B = 1 for baryons / B = -1 for anti-baryons / B = 0 others

Why do we observe only the baryon/meson combinations only?

Why can we observe such particles as the Δ++ = (u↑ u↑ u↑) forbidden by the Fermi statistics?

➔ Because of the colored charge : the only allowed (physics) states correspond to “white” combinations of quarks and antiquarks.

1.3 Hadrons

3 basic colors: **R G B** for the quarks and their "anti"-colors for the anti-quarks : $\bar{\mathbf{R}}$ $\bar{\mathbf{G}}$ $\bar{\mathbf{B}}$

White combinations correspond to :

- **RGB** or $\bar{\mathbf{R}}\bar{\mathbf{G}}\bar{\mathbf{B}}$ in equal proportions
- $\mathbf{R}\bar{\mathbf{R}}$ $\mathbf{G}\bar{\mathbf{G}}$ $\mathbf{B}\bar{\mathbf{B}}$ in equal proportions
  (where "proportions" means the proper anti-symmetrization)

Example: "white proton" p (u u d)

1.3 Hadrons

A more exhaustive list… with only a few examples

Mésons sans saveurs $L_e, L_\mu, L_\tau, B, S, C, \widetilde{B}, T = 0, I_3 = Q$ $I = 1\ (\pi, b, \rho, a) : u\bar{d}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), d\bar{u}$ $I = 0\ (\eta, \eta', h, h', \omega, \phi, f, h) : c_1(u\bar{u} + d\bar{d}) + c_2 s\bar{s}$						
	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$\pi^\pm$	139.57018(35)	$\tau = 2.60330(5) \times 10^{-8}$ s	± 1	$(0,0,0,0)$	1^-	0^-
π^0	134.9766(6)	$\tau = 8.4(6) 10^{-17}$ s	0	$(0,0,0,0)$	1^-	0^{-+}
η	547.75(12)	0.00129(7)	0	$(0,0,0,0)$	0^+	0^{-+}
$\eta'(958)$	957.78(14)	0.202(16)	0	$(0,0,0,0)$	0^+	0^{-+}
$\rho(770)$	775.8(5)	150.3(16)	$\pm 1, 0$	$(0,0,0,0)$	1^+	1^{--}
$\phi(1020)$	1019.456(20)	4.26(5)	0	$(0,0,0,0)$	0^-	1^{--}
$\omega(782)$	782.59(11)	8.49(8)	0	$(0,0,0,0)$	0^-	1^{--}

Mésons beaux ($\widetilde{B} = \pm 1$) $L_e, L_\mu, L_\tau, B, S, \widetilde{B}, T = 0, I_3 = Q - \frac{1}{2}\widetilde{B}$ $B^+ = u\bar{b},\quad B^0 = d\bar{b},\quad \bar{B}^0 = \bar{u}b,\quad B^- = \bar{u}b$						
	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$B^\pm$	5279.0 (5)	$\tau = 1.671(18) \times 10^{-12}$ s	± 1	$(0,0,\pm 1,0)$	$\frac{1}{2}$	0^-
$B^0, \bar{B}^0$	5279.4 (5)	$\tau = 1.536(14) \times 10^{-12}$ s	0	$(0,0,\pm 1,0)$	$\frac{1}{2}$	0^-
B^*	5325.0 (6)		$\pm 1, 0$	$(0,0,0,0)$	$\frac{1}{2}$	1^-

1.3 Hadrons

A more exhaustive list… with only a few examples

Mésons étranges $L_e, L_\mu, L_\tau, B, C, \widetilde{B}, T = 0, I_3 = Q - \frac{1}{2} S$ $K^+ = u\bar{s},\ K^0 = d\bar{s},\ \bar{K}^0 = \bar{d}s,\ K^- = \bar{u}s$ $K^{*+} = u\bar{s},\ K^{*0} = d\bar{s},\ \bar{K}^{*0} = \bar{d}s,\ K^{*-} = \bar{u}s$						
	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$K^\pm$	493.677(16)	$\tau = 1.2384(24) \times 10^{-8}$ s	± 1	$(\pm 1, 0, 0, 0)$	$\frac{1}{2}$	0^-
$K^0, \bar{K}^0$	497.648(22)		0	$(\pm 1, 0, 0, 0)$	$\frac{1}{2}$	0^-
K_S^0		$\tau = 0.8958(6) \times 10^{-10}$ s	0	$(\ , 0, 0, 0)$	$\frac{1}{2}$	0^-
K_L^0		$\tau = 5.18(4) 10^{-8}$ s	0	$(\ , 0, 0, 0)$	$\frac{1}{2}$	0^-
$K^*(892)$	891.66(26)	50.8(9)	$\pm 1, 0$	$(\pm 1, 0, 0, 0)$	$\frac{1}{2}$	1^-
$K^{*0}, \bar{K}^{*0}(892)$	896.10(27)	50.7(6)	0	$(\pm 1, 0, 0, 0)$	$\frac{1}{2}$	1^-

Mésons étranges charmés $S = C = \pm 1$ $L_e, L_\mu, L_\tau, B, S, \widetilde{B}, T, I_3 = 0$ $D_s^+ = c\bar{s},\ D_s^- = \bar{c}d,\ D_s^{*+} = c\bar{s},\ D_s^{*-} = \bar{c}d$						
	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$D_s^\pm$	1968.3(5)	$\tau = 490\ (9) \times 10^{-15}$ s	± 1	$(\pm 1, \pm 1, 0, 0)$	0	0^-
$D_s^{*\pm}$	2112.1(7)	< 1.9	± 1	$(\pm 1, \pm 1, 0, 0)$	0	

Mésons $c\bar{c}$ $L_e, L_\mu, L_\tau, B, S, C, \widetilde{B}, T, I_3, Q = 0$ $c\bar{c}$						
	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$\eta_c(1S)$	2979.6 (12)	$17.3^{(+27)}_{(-25)}$	0	$(0, 0, 0, 0)$	0^+	0^{-+}
$J/\Psi(1S)$	3096.016 (11)	91.0 (32)	0	$(0, 0, 0, 0)$	0^-	1^{--}

1.3 Hadrons

A more exhaustive list… with only a few examples

Baryons $(S=0, I=\frac{1}{2})$

$L_e, L_\mu, L_\tau, S, C, \widetilde{B}, T = 0, B = 1, I_3 = Q - \frac{1}{2}$

$p, N^+ = uud, \quad n, N^0 = udd$

	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
p	938.27203(8)	$\tau > 10^{31} - 10^{33}$ ans	$+1$	$(0,0,0,0)$	$\frac{1}{2}$	$\frac{1}{2}^+$
n	939.56536(8)	$\tau = 885.7\ (8)$ s	0	$(0,0,0,0)$	$\frac{1}{2}$	$\frac{1}{2}^+$
$N(1440)\ P_{11}$	$1430 - 1470$	$250 - 450$	$+1, 0$	$(0,0,0,0)$	$\frac{1}{2}$	$\frac{1}{2}^+$
$N\ (1520)\ D_{13}$	$1515 - 1530$	$110 - 135$	$+1, 0$	$(0,0,0,0)$	$\frac{1}{2}$	$\frac{3}{2}^-$

Baryons $\Delta\ (S=0, I=\frac{3}{2})$

$L_e, L_\mu, L_\tau,\ S, C, \widetilde{B}, T = 0, B = 1, I_3 = Q - \frac{1}{2}$

$\Delta^{++} = uuu, \quad \Delta^+ = uud, \quad \Delta^0 = udd, \quad \Delta^- = ddd$

	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
$\Delta(1232)\ P_{33}$	$1232 - 1234$	$115 - 125$	$+2, +1, 0 - 1$	$(0,0,0,0)$	$\frac{3}{2}$	$\frac{3}{2}^+$
$\Delta(1600)\ P_{33}$	$1550 - 1700$	$250 - 450$	$+2, +1, 0 - 1$	$(0,0,0,0)$	$\frac{3}{2}$	$\frac{3}{2}^+$

Baryons $\Lambda\ (S=-1, I=0)$

$L_e, L_\mu, L_\tau, C, \widetilde{B}, T, I_3, Q = 0, B = 1$

$\Lambda^0 = uds$

	M (MeV)	Γ (MeV) ou τ	Q	$(S, C, \widetilde{B}, T)$	I^G	J^{PC}
Λ	1115.683 (6)	$\tau = 2.632\ (20) \times 10^{-10}$ s	0	$(-1,0,0,0)$	0	$\frac{1}{2}^+$
$\Lambda\ (1405)\ S_{01}$	1406 (4)	50.0 (20)	0	$(-1,0,0,0)$	0	$\frac{1}{2}^-$

2. Hadron spectroscopy

Introduction:

- 1st observation : invariance of the strong interactions w.r.t. the electric charge (*p-p*, *p-n*, *n-n* are equivalent for the strong interactions: underlying symmetry?)
- 2nd observation : masses identity

$$m_P = 938.3\,(\text{MeV}) \quad m_N = 939.1\,(\text{MeV})$$
$$m_{\pi^\pm} = 139.6\,(\text{MeV}) \quad m_{\pi^0} = 135.0\,(\text{MeV})$$
$$m_{K^\pm} = 493.7\,(\text{MeV}) \quad m_{K^0} = 497.7\,(\text{MeV})$$

Those masses would have been probably degenerate in absence of e.m. interactions (symmetry violation analog to the Zeeman effect)

2.1 Isospin symmetry

Generalization to multiplets :

Multiplet $J^P = 0^-$			Multiplet $J^P = 1^-$		
Mésons	Mass(MeV)	Nom	Mésons	Mass(MeV)	Nom
π^+, π^0, π^-	$139.6, 135.0, 139.6$	pion	ρ^+, ρ^0, ρ^-	768.5	rho
K^+, K^0	$493.7, 497.7$	kaon	ω	781.9	oméga
$\bar{K}^0, K^-$	$497.7, 493.7$	antikaon	K^{*+}, K^{*0}	$891.6, 896.1$	kaon étoile
η	547.5	eta	$\bar{K}^{*0}, K^{*-}$	$896.1, 891.6$	antikaon étoile
η'	957.8	eta prime	ϕ	1019.4	phi

Multiplet $J^P = \frac{1}{2}^+$			Multiplet $J^P = \frac{3}{2}^+$		
Baryons	Mass(MeV)	Nom	Baryons	Mass(MeV)	Nom
p, n	$938.3, 939.6$	nucléon	$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	≈ 1232	delta
Λ	1115.7	lambda	$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	$1382.8, 1383.7, 1387.2$	sigma étoile
$\Sigma^+, \Sigma^0, \Sigma^-$	$1189.4, 1192.6, 1197.4$	sigma	Ξ^{*0}, Ξ^{*-}	$1530.8, 1535.0$	xi étoile
Ξ^0, Ξ^-	$1314.9, 1321.3$	xi	Ω^-	1672.5	oméga

2.1 Isospin symmetry

Conclusion: hadrons can be classified as multiplets of ~equal masses particles differing by their electrical charge :

singulets : $\eta, \eta', \omega, \phi, \Lambda, \Omega^-$

doublets : $\begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}, \begin{pmatrix} K^{*+} \\ K^{*0} \end{pmatrix}, \begin{pmatrix} \bar{K}^{*0} \\ K^{*-} \end{pmatrix}, \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$

triplets : $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}, \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}, \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} \\ \Sigma^{*0} \\ \Sigma^{*-} \end{pmatrix}$

quadruplets : $\begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$

From the point of view of strong interactions, proton and neutron are almost the same particle. One creates an abstract space in which strong interactions are invariant under rotations…

2.1 Isospin symmetry

The conserved quantity (Noether's theorem) is called isospin.

Isospin treatment follows the kinetic moment one's and relies on the group theory (SU(2) representation) :

- In the isospin space one introduces an operator $\vec{I} = (I_1, I_2, I_3)$ which commutation rules read $\left[I_i, I_j \right] = \varepsilon_{ijk} I_k$
- The eigenstates $\left| I, I_3 \right\rangle$ of the observables I^2, I_3 are such that :

$$I^2 \left| I, I_3 \right\rangle = I(I+1) \left| I, I_3 \right\rangle$$

$$I_3 \left| I, I_3 \right\rangle = I_3 \left| I, I_3 \right\rangle$$

- A multiplet has $2I+1$ eigenstates $I_3 = -I, \ldots, +I$ ("2I+1"-plet)

2.1 Isospin symmetry

Within such representations one has the following multiplets:

$$|p\rangle = \left|I = \frac{1}{2}, I_3 = \frac{1}{2}\right\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$$

$$|n\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle,$$

Doublet

$$|\Delta^{++}\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle$$

$$|\Delta^{+}\rangle = \left|\frac{3}{2}, \frac{1}{2}\right\rangle$$

$$|\Delta^{0}\rangle = \left|\frac{3}{2}, -\frac{1}{2}\right\rangle$$

$$|\Delta^{-}\rangle = \left|\frac{3}{2}, -\frac{3}{2}\right\rangle$$

Quadruplet

2.1 Isospin symmetry

Let's define an operator acting on the isospin multiplet

$$|\Phi'\rangle = U|\Phi\rangle$$

- U must be a unitary complex matrix $U^{\dagger}U = 1$
- unimodular $\det(U) = 1$
- of rank 1 (only 1 matrix simultaneously diagonalizable)

➔ Those matrices are known : the 2×2 special unitary matrices from SU(2) group (spin representation)

2.2 Basics of Lie groups

SU(N) groups :

- Group generators : I_k defined such as $U(\alpha_1,\alpha_2,\alpha_3) = e^{-i\alpha_k I_k}$
  \# of independent generators : $m = n^2 - 1$
- Properties : ${I_k}^{\dagger} = I_k$ and $Tr(I_k) = 0$
- \# of generators simultaneously diagonalizable (rank) : $r = n - 1$
- \# of Casimir operators (function of the generators commuting with all of them) : $r = n - 1$
  ex. SU(2) : $I^2 = {I_1}^2 + {I_2}^2 + {I_3}^2$
- The fundamental representation is of dimension N : $\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{pmatrix}$
- Structure constants defined by: $\left[I_i, I_j\right] = f_{ijk} I_k$

2.2 Basics of Lie groups

SU(2) (spin-isospin…)

- Generators proportional to the Pauli matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Fundamental representation (doublet) : each state can be build on basis vectors

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = X_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + X_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Structure constants $f_{ijk} = \varepsilon_{ijk}$ (permutation operator)

2.2 Basics of Lie groups

Product of fundamental representations:

- Like the spin couplings

$$\left|I_a, I_b, I, I_3\right\rangle = \sum_{I_{a3}, I_{b3}} C(I_a, I_b, I, I_3)\left|I_a, I_{a3}, I_b, I_{b3},\right\rangle$$

- Example in SU(2): $\frac{\vec{1}}{2}+\frac{\vec{1}}{2}=\vec{0}+\vec{1}$ or $2\otimes 2 = 1\oplus 3$

- Generation of an isospin triplet : example of pions:

$$\left|\pi^+\right\rangle \equiv \left|1,1\right\rangle = \left|\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$\left|\pi^0\right\rangle \equiv \left|1,0\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2},\tfrac{-1}{2}\right\rangle + \left|\tfrac{1}{2},\tfrac{1}{2};\tfrac{-1}{2},\tfrac{1}{2}\right\rangle\right)$$

$$\left|\pi^-\right\rangle \equiv \left|1,-1\right\rangle = \left|\tfrac{1}{2},\tfrac{1}{2};\tfrac{-1}{2},\tfrac{-1}{2}\right\rangle$$

- + a remaining isospin singlet

2.2 Basics of Lie groups

Gell-Mann Nishijima relation :

- Strong interactions are invariant under a rotation around the 3rd component of the isospin
- I_3 value is also a measurement of the charge eigenstate inside the multiplet

➔ There should be a link between I_3 and Q

$$Q = I_3 + Y/2 \quad (Y = B + s + c + b + t)$$

Invariant through all interactions (→ Q)

Invariant through strong Interactions only (→ I_3)

Hypercharge (→ Y)

2.2 Basics of Lie groups

Adjoint representation:

- In SU(2), *p* and *n* form an isospin doublet, what about $\bar{p}$ et $\bar{n}$?
  In isospin space let's take a rotation around the 2nd axis :

$$U(\theta) = 1 - i\theta\tfrac{\tau_2}{2} + \tfrac{1}{2!}\left(i\theta\tfrac{\tau_2}{2}\right)^2 + \ldots = \cos\tfrac{\theta}{2}.1 - i\sin\tfrac{\theta}{2}.\tau_2$$

- Rotated states : $\begin{pmatrix} p' \\ n' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$

- Gell-Mann Nishijima relation: $Q(\bar{p}) = -Q(p) = -1 \quad Q(\bar{n}) = 0$

$$B(\bar{p}) = B(\bar{p}) = -1$$

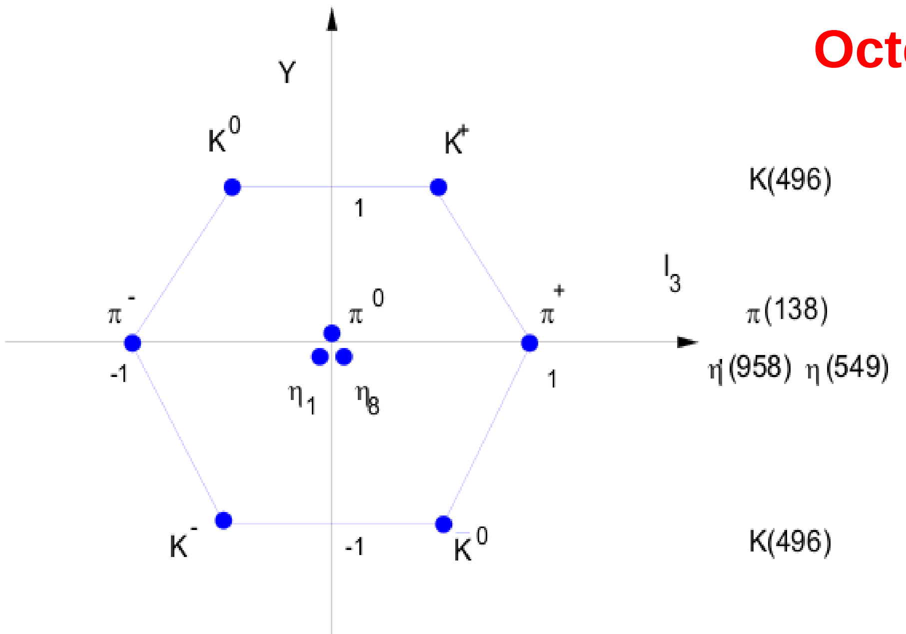
$$\Rightarrow I_3(\bar{p}) = -\tfrac{1}{2} \quad I_3(\bar{n}) = +\tfrac{1}{2}$$

- Adjoint representation: $\begin{pmatrix} \bar{n}' \\ -\bar{p}' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \bar{n} \\ -\bar{p} \end{pmatrix}$

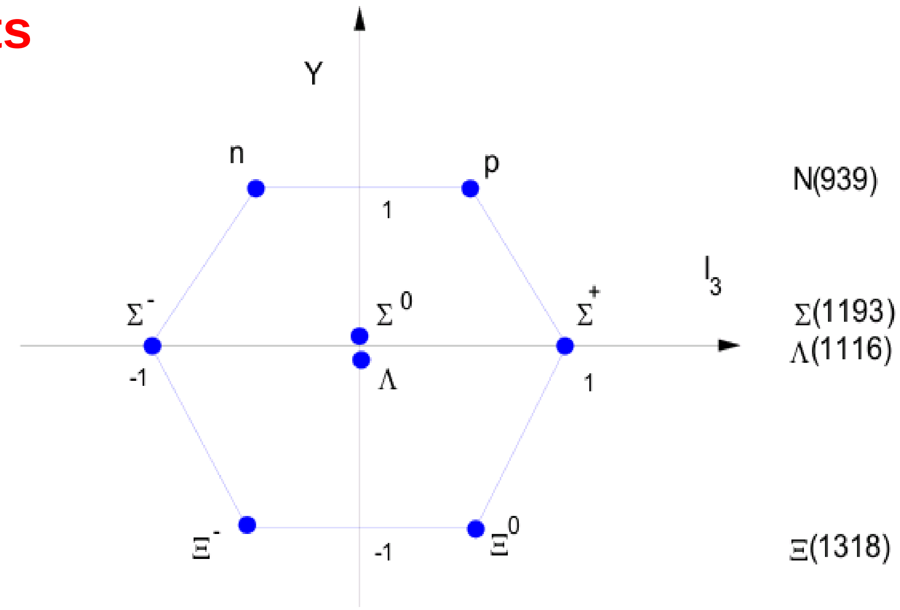
2.3 Quarks model

- The adjoint representation is generated by the following matrices: $(I_i)_{jk} = i\, f_{ijk}$

- Quark model: each hadron multiplet is characterized by a baryonic number, a spin and a defined parity or equivalently by an hypercharge and the 3rd component of the isospin :



Octets



2.3 Quarks model

- In a constant-Y row the masses are nearly equal. Gell-Mann and Zweig proposed in 1964 that hadrons were not elementary but compounds of 3 types of quarks : u, d, s.
- In terms of group theory this model will be read through SU(3) representations. To be complete the model has to be extended to 6 flavors (including c, b, t) and convoluted with a space of colors (3 colors: R G B, already mentioned)
- The SU(3) group. Triplet fundamental representation : $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$.

Basis vectors :

	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
I_3	$\frac{1}{2}$	$-\frac{1}{2}$	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$

2.3 Quarks model

- Gell-Mann matrices (8 generators)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- Structure constants

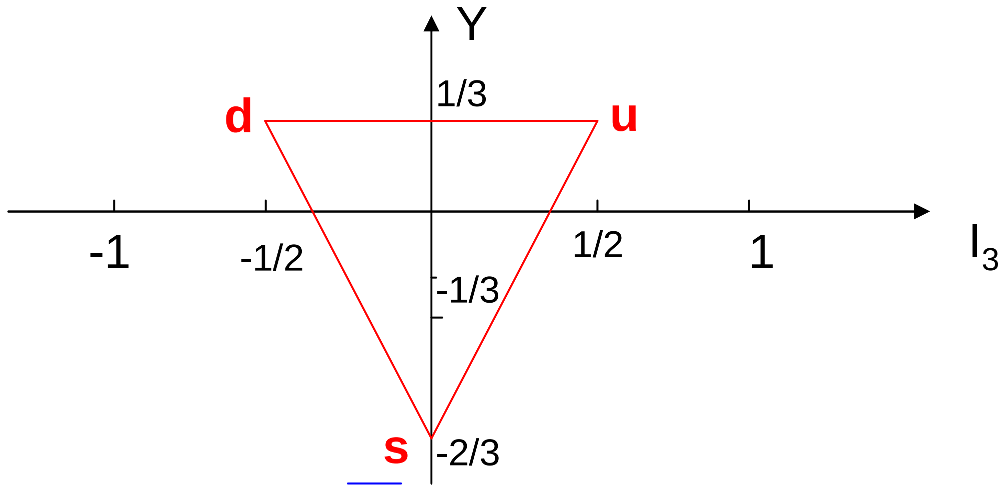
$$f_{123} = 1$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

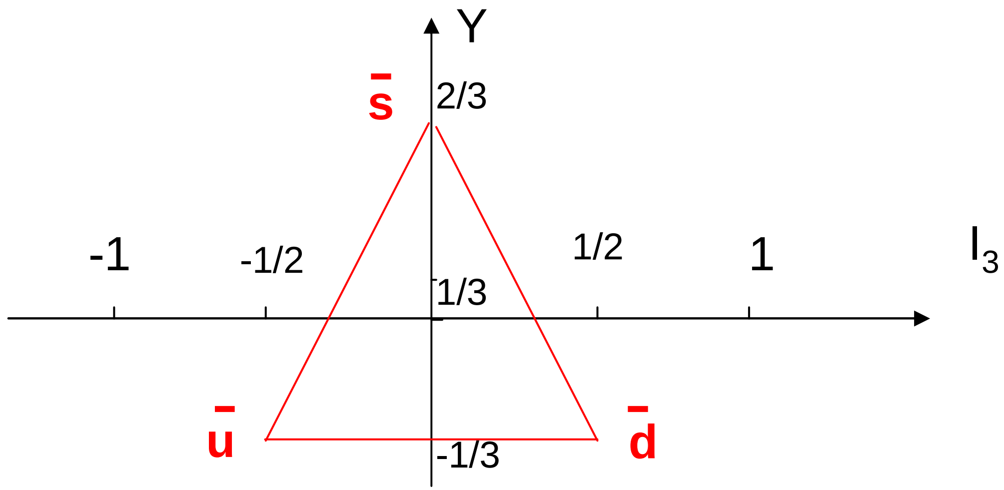
$$f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}$$

2.3 Quarks model

- Fundamental representation 3 :



- Adjoint representation $\bar{3}$:



2.3 Quarks model

- How do we generate the hadron spectrum?

- For mesons : $3 \otimes \overline{3} = 1 \oplus 8$ (singlet + octet)

- For baryons : $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8' \oplus 10$(singlet + octets + decuplet)

- The construction of the wavefunctions is rather technical (proper antisymmetrization etc) but the principle is there…

2.3 Quarks model

- Examples in the mesonic sector
- Quark contents:

Octet :

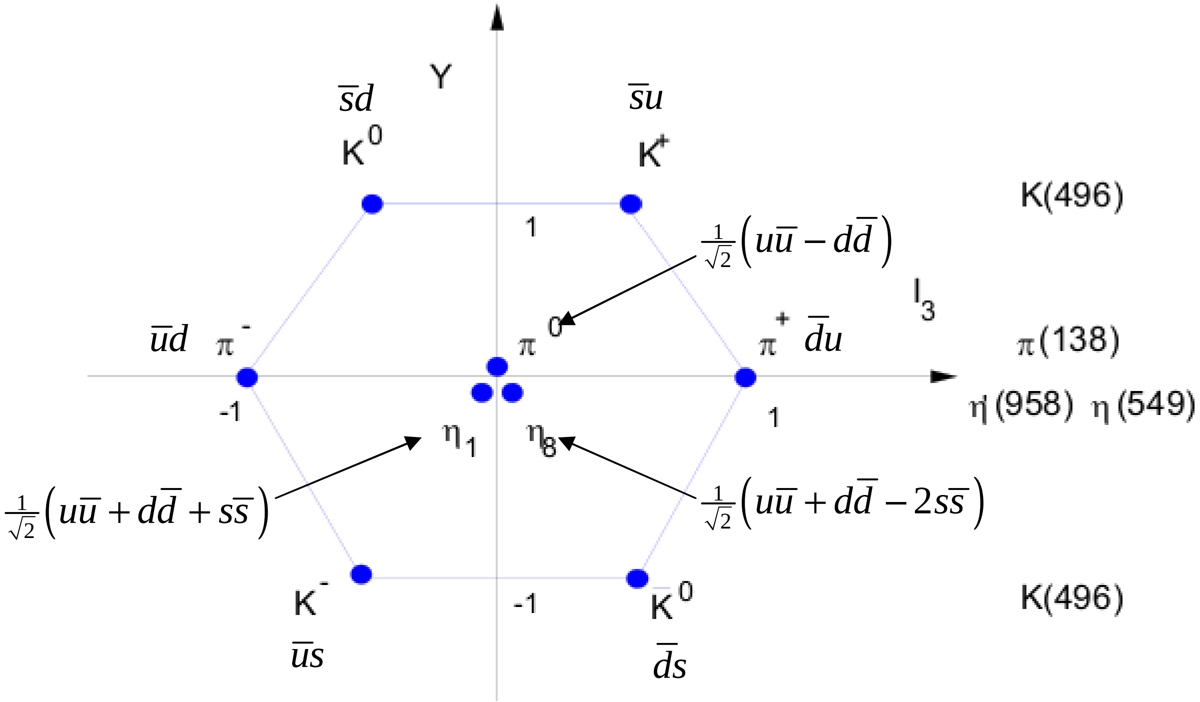
quark-antiquark dans $SU(3)$, états mixtes	
Constituants	$\mathbf{8}_M$
$\pi^+(ud)$	ud
$\pi^0(u\bar{u}, d\bar{d})$	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$\pi^-(d\bar{u})$	$d\bar{u}$
$K^+(u\bar{s})$	$u\bar{s}$
$K^0(d\bar{s})$	$d\bar{s}$
$\bar{K}^0(s\bar{d})$	$s\bar{d}$
$K^-(s\bar{u})$	$s\bar{u}$
$\eta_8(u\bar{u}, d\bar{d}, s\bar{s})$	$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$

Singlet :

quark-antiquark dans $SU(3)$, états antisymétriques	
Constituants	$\mathbf{1}_A$
$\eta_1(u\bar{u}, d\bar{d}, s\bar{s})$	$\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$

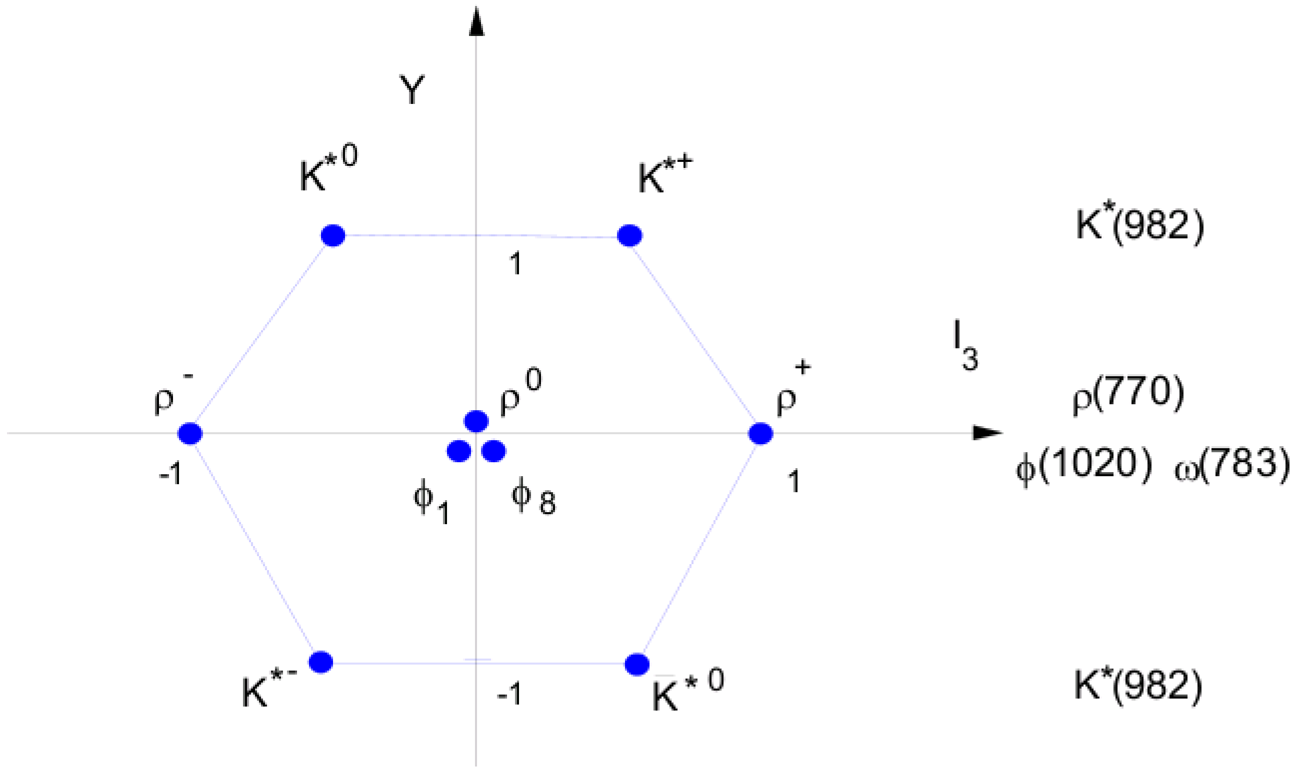
2.3 Quarks model

- Pseudo-scalar mesons diagram (octet + singlet): $J^P = 0^-$



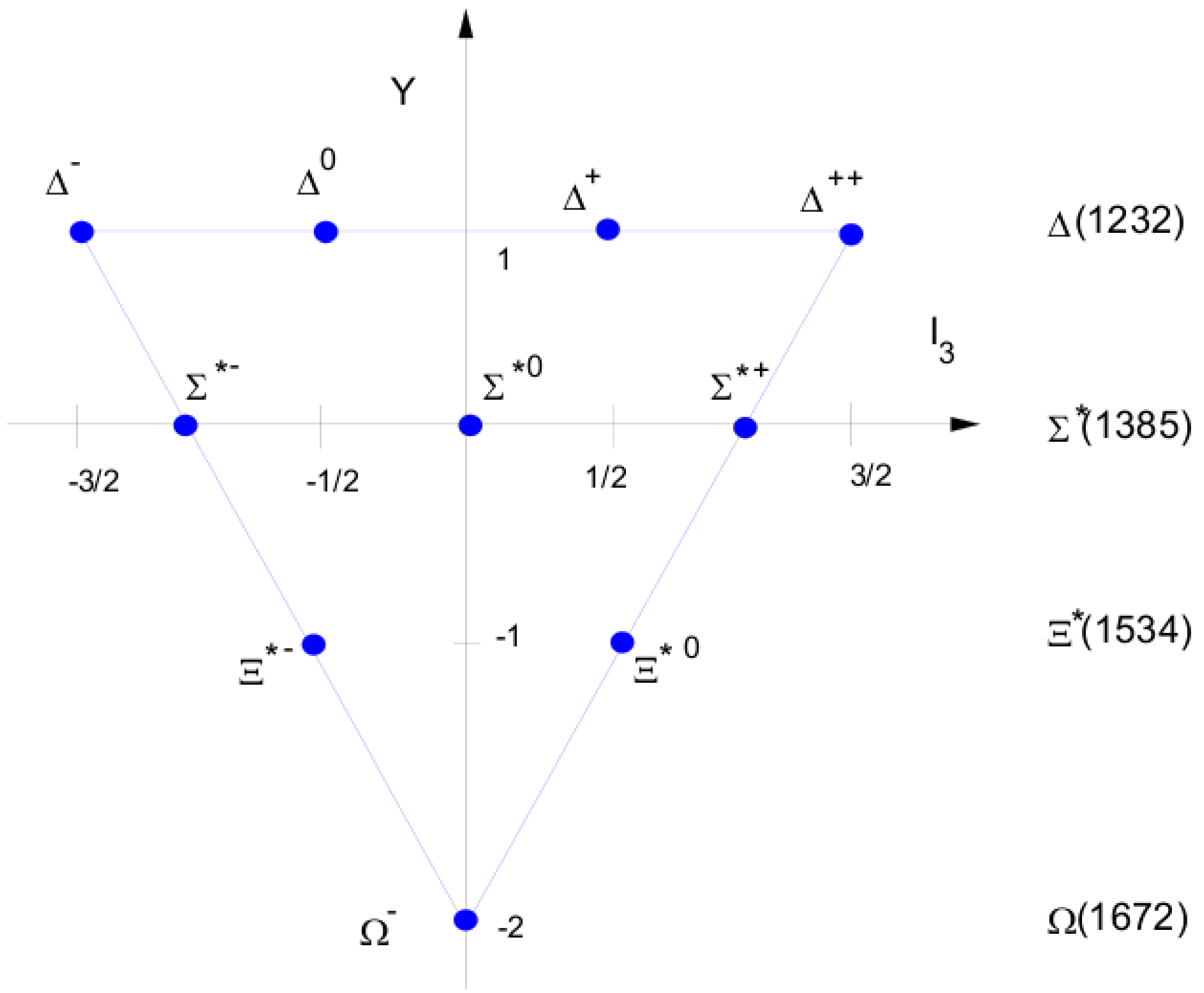
2.3 Quarks model

- Pseudo-vector mesons diagram (octet + singlet): same quarks content but // spin : $J^P = 1^-$



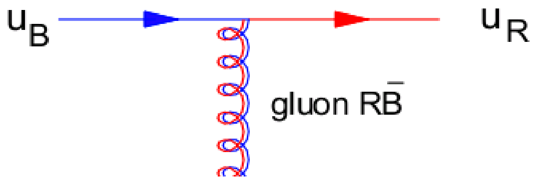
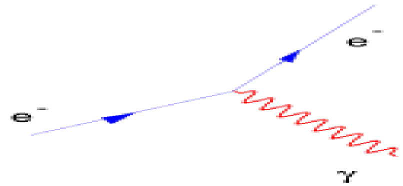
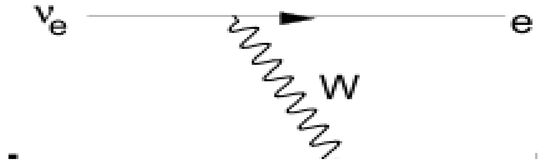
2.3 Quarks model

- Just for fun : baryons decuplet



3- Fundamental interactions

- Leptons and quarks interactions are mediated by specific gauge bosons :

strong	gluons	g (8)	M=0	
e.m.	photon	γ (1)	M=0	
weak	Z^0 $W^{\pm}$		M=90 GeV M=81 GeV	
gravitation	graviton	$h^{\mu\nu}$	?	?

3.1- Range and propagators

- Yukawa approach: in 1935 it was proposed a link between the range of an interaction and the mass of the "carrier" quantum.
- Heisenberg inegality :

$$R = c\Delta t \approx 1/\Delta E = 1/m$$

- Formally (Klein-Gordon equation ie "massive" photon propagation equation): $\Box\psi + m^2\psi = 0$

  For a static spherical potential :

$$\triangle\tilde{\psi}(r) + m^2\tilde{\psi}(r) = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\tilde{\psi}(r)\right)\right) + m^2\tilde{\psi}(r) = 0$$

$$\Rightarrow \tilde{\psi}(r) = \frac{g}{4\pi r}e^{-r/R} \quad \text{where} \quad R = \frac{1}{m}$$

3.1- Range and propagators

- Historically this approach led to the prediction of an intermediate quantum for the strong interaction of mass close to:

$$m = 1/R \sim 1/\text{few fm} \sim 100 - 200\,\text{MeV}$$

- The pion was then discovered (140 MeV) which can be seen as the carrier for the residual strong interaction between nucleons (not quarks) at the scale of the nucleus.

- Its small mass is the manifestation of another symmetry breaking, the chiral symmetry $SU(2)_L \times SU(2)_R$.

- This simple model gives an idea of the range of an interaction and the link with the intermediate properties.

3.1- Range and propagators

- The Fourier transform of the Yukawa potential is given by :

$$\tilde{\psi}(r) \propto \frac{e^{-r/R}}{r} \quad \text{where} \quad R = \frac{1}{m}$$

$$\Rightarrow \tilde{\psi}(q) \propto \frac{1}{\vec{q}^2 + m^2 - i\varepsilon}$$

- The quantity obtained is called the propagator.
- Using the Green function formalism for the Klein-Gordon equation: $(\Box + m^2)\psi(x) = 0 \quad \text{where} \quad x = x^\mu = (x^0, \vec{x})$

$$\Rightarrow (q^2 - m^2)\psi(q) = 0 \quad \text{with} \quad q = q^\mu = (q^0, \vec{q})$$

$$\Rightarrow (q^2 - m^2)G(q) = \delta^4(q)$$

$$\Rightarrow G(q) = \frac{\delta^4(q)}{q^2 - m^2} \Rightarrow \boxed{\text{Propagator} = \frac{i}{q^2 - m^2}}$$

3.1- Range and propagators

- What is the role of the integration constant g?

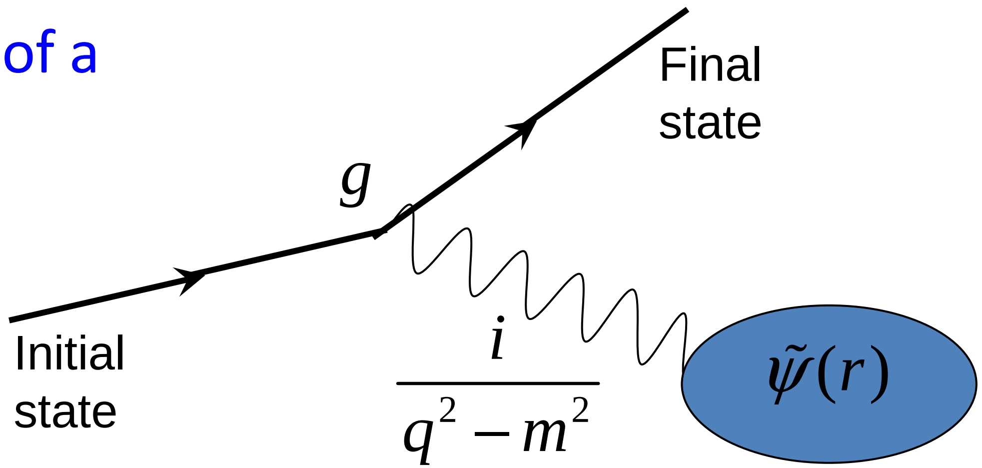
$$\tilde{\psi}(r) = \frac{g}{4\pi r} e^{-r/R}$$

- In Electromagnetism we have the standard Coulomb potential :

$$\tilde{\psi}(r) = \frac{Q}{4\pi r}$$

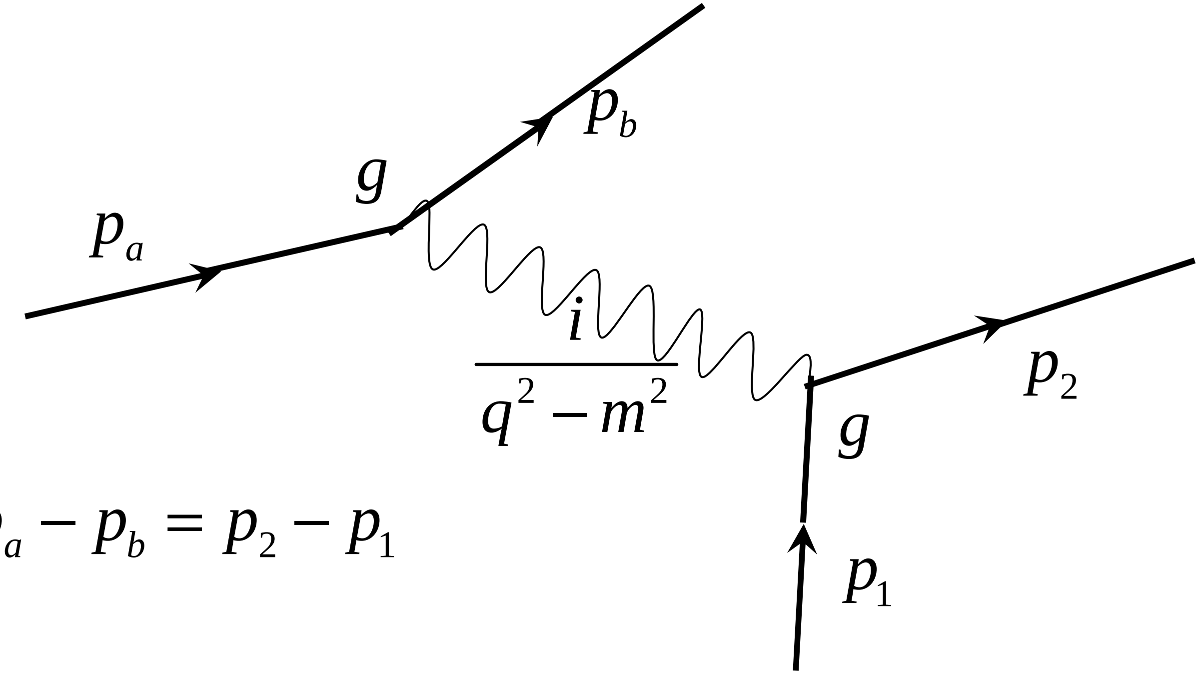
corresponding to a vanishing mass (for photon) or infinite range

- *Q* and *g* play the role of a coupling constant for the interaction



3.1- Range and propagators

- Basic Feynman diagram :



$$q = p_a - p_b = p_2 - p_1$$

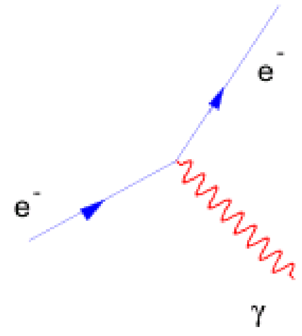
- The amplitude of the process writes:

$$g^2 A(p_a, p_b) \frac{1}{q^2 - m^2} A(p_1, p_2) \delta^4(p_a + p_1 - p_b - p_2)$$

3.2- Electroweak interactions

ELECTROMAGNETIC INTERACTIONS

- Coupling constant : $\alpha_{em} = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$
- Example : Rutherford cross-section $\frac{\partial\sigma}{\partial q^2} \propto \frac{\alpha^2{}_{em}}{q^4}$

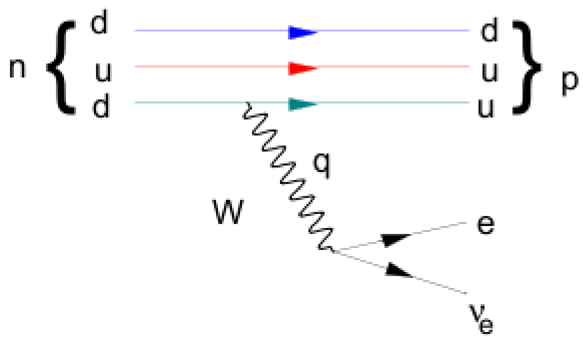


- Typical cross-section $\sim 10^{-33}\ \mathrm{m}^2$
- Typical interaction times $\sim 10^{-20}\ \mathrm{s}$
- Photon (γ) exchange, infinite range

3.2- Electroweak interactions

WEAK INTERACTIONS

- Coupling constant : $\alpha_{Fermi} = \dfrac{G_F {m_P}^2}{4\pi} \approx 10^{-6}$
- Example : neutron β-decay

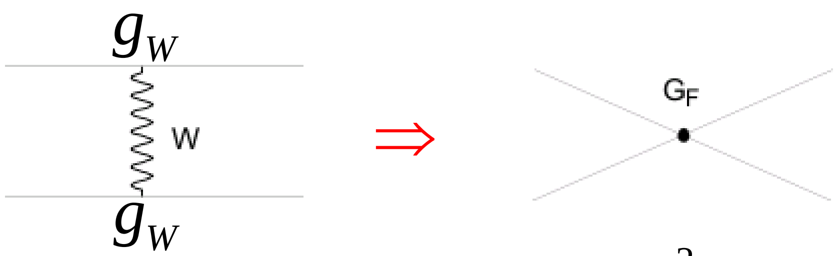


- Weak interactions do not conserve the (
- Typical cross-section $\sim 10^{-44}\ \mathrm{m}^2$
- Typical interaction times $\sim 10^{-10}\ \mathrm{s}$
- Weak bosons exchange, finite range

3.2- Electroweak interactions

WEAK INTERACTIONS (cont'd)

- Typical range : $M \sim 80-90\ \text{GeV} \Rightarrow R \sim 10^{-18}\ \text{m}$
- Due to the large mass of the exchanged bosons the weak interactions can often be considered point-like



$$g_W{}^2 \frac{1}{q^2 - M_W{}^2} \xrightarrow[q^2 \to 0]{} \frac{g_W{}^2}{M_W{}^2} \equiv G_F = 10^{-5}\ \text{GeV}^{-2}$$

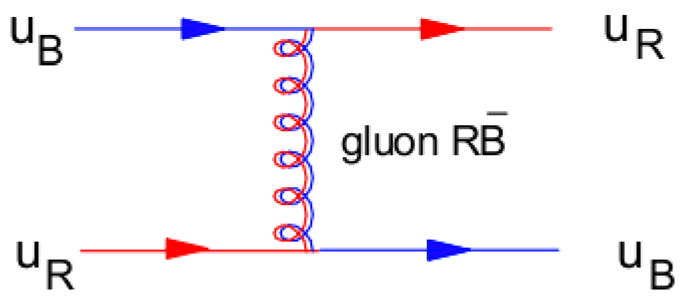
- Weinberg angle (electroweak theory) :

$$\alpha_W = \frac{g_W{}^2}{4\pi} = \frac{e^2}{4\pi \sin^2\theta_W} = \frac{1}{29} \quad (\sin^2\theta_W = 0.22)$$

3.3- Strong interactions

Features

- Coupling constant : $\alpha_s \simeq 1$

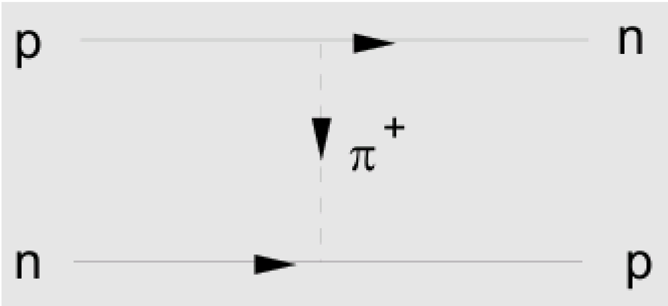


- Carry a colored charge
- Typical cross-section $\sim 10^{-30}\ \mathrm{m}^2$
- Typical interaction times $\sim 10^{-23}\ \mathrm{s}$
- Gluons (g) exchange, effective finite range due to the confinement $R \sim 10^{-15}\,\mathrm{m}$. Asymptotic freedom.

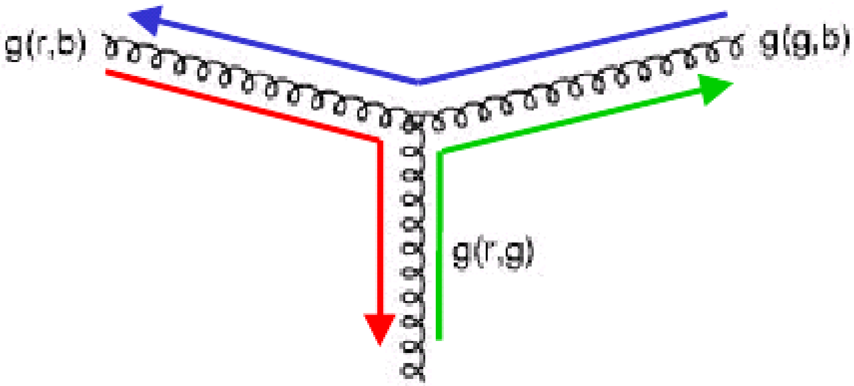
3.3- Strong interactions

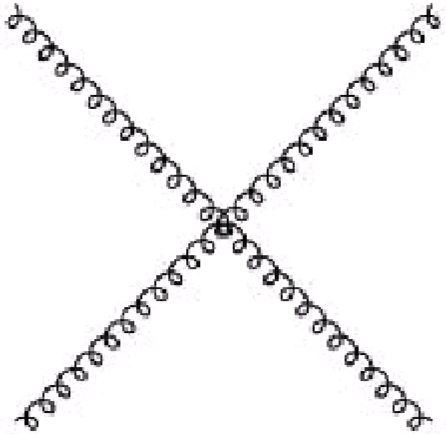
Features (cont'd)

- Residual interaction (at the nuclei scale) :



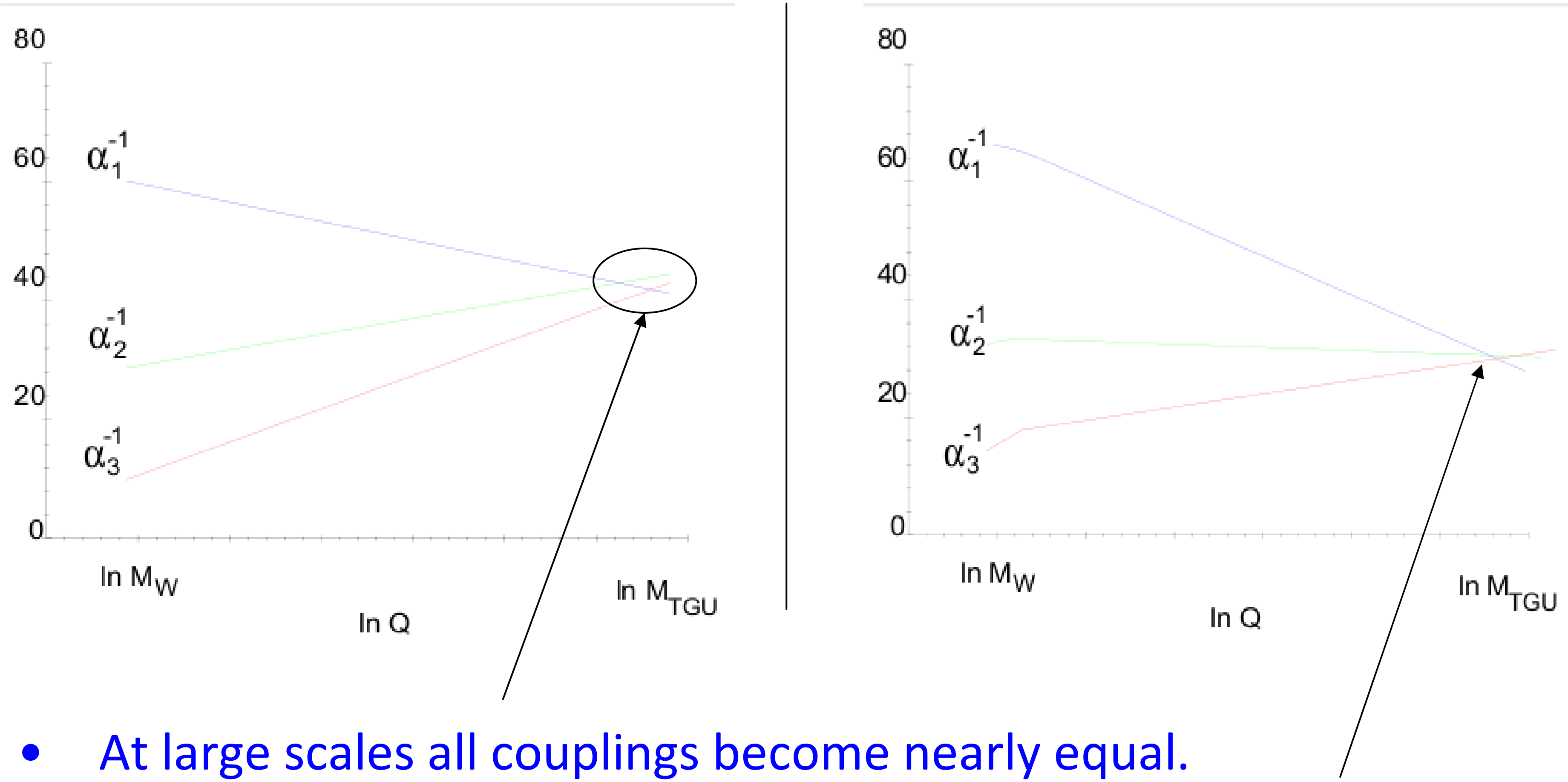
- ∃ couplings between gluons :





3.3- Towards unification?

- Coupling constants vary with the energy



- At large scales all couplings become nearly equal.
- In some “beyond Standard Model” models (SUSY etc) this occurs!